Higher

Mathematics

UNIT 2 OUTCOME 3

Trigonometry

Contents

HSN22300

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OUTCOME 3

Trigonometry

1 Solving Trigonometric Equations

You should already be familiar with solving some trigonometric equations.

EXAMPLES

1. Solve $\sin x° = \frac{1}{2}$ for $0 < x < 360$.

2. Solve $\cos x° = -\frac{1}{\sqrt{5}}$ for $0 < x < 360$.

3. Solve $\sin x° = 3$ for $0 < x < 360$.

4. Solve $\tan x° = -5$ for $0 < x < 360$.

Note

All trigonometric equations we will meet can be reduced to problems like those above. The only differences are:

- the solutions could be required in radians – in this case, the question will not have a degree symbol, e.g. "Solve $3\tan x = 1$" rather than "$3\tan x° = 1$";
- exact value solutions could be required in the non-calculator paper – you will be expected to know the exact values for 0, 30, 45, 60 and 90 degrees.

Questions can be worked through in degrees or radians, but make sure the final answer is given in the units asked for in the question.

EXAMPLES

5. Solve $2\sin 2x° - 1 = 0$ where $0 \le x \le 360$.

6. Solve $\sqrt{2}\cos 2x = 1$ where $0 \leq x \leq \pi$.

7. Solve $4\cos^2 x = 3$ where $0 < x < 2\pi$.

8. Solve $3\tan(3x° - 20°) = 5$ where $0 \leq x \leq 360$.

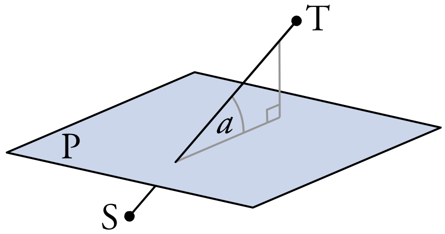
9. Solve $\cos\left(2x+\frac{\pi}{3}\right)=0{\cdot}812$ for $0<x<2\pi$.

2 Trigonometry in Three Dimensions

It is possible to solve trigonometric problems in three dimensions using techniques we already know from two dimensions. The use of sketches is often helpful.

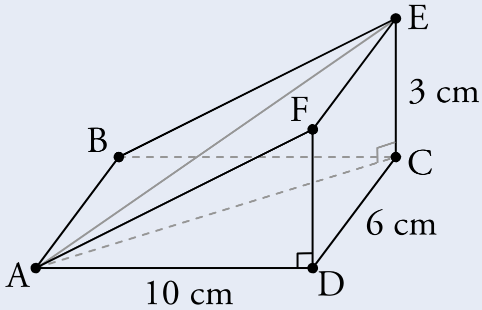
The angle between a line and a plane

The angle a between the plane P and the line ST is calculated by adding a line perpendicular to the plane and then using basic trigonometry.



EXAMPLE

1. The triangular prism ABCDEF is shown below.



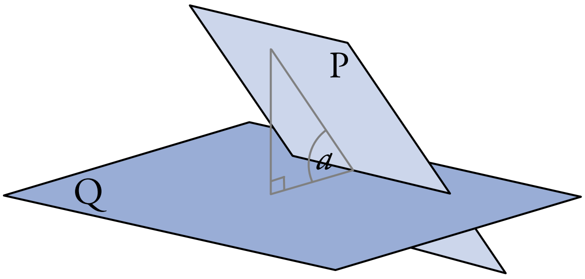
Calculate the acute angle between:

(a) The line AF and the plane ABCD.

(b) AE and ABCD.

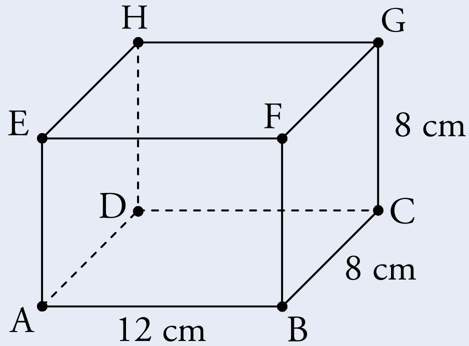
The angle between two planes

The angle a between planes P and Q is calculated by adding a line perpendicular to Q and then using basic trigonometry.



EXAMPLE

2. ABCDEFGH is a cuboid with dimensions $12 \times 8 \times 8$ cm as shown below.



(a) Calculate the size of the angle between the planes AFGD and ABCD.

(b) Calculate the size of the acute angle between the diagonal planes AFGD and BCHE.

3 Compound Angles

When we add or subtract angles, the result is called a **compound angle**.

For example, $45° + 30°$ is a compound angle. Using a calculator, we find:

- $\sin(45° + 30°) = \sin(75°) = 0{\cdot}966$;
- $\sin(45°) + \sin(30°) = 1{\cdot}207$ (both to 3 d.p.).

This shows that $\sin(A + B)$ is *not* equal to $\sin A + \sin B$. Instead, we can use the following identities:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B.$$

These are given in the exam in a condensed form:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

EXAMPLES

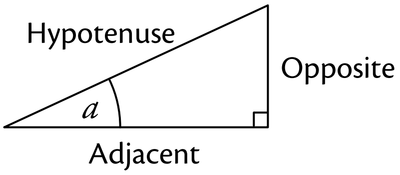
1. Expand and simplify $\cos(x° + 60°)$.

2. Show that $\sin(a + b) = \sin a \cos b + \cos a \sin b$ for $a = \frac{\pi}{6}$ and $b = \frac{\pi}{3}$.

3. Find the exact value of $\sin 75°$.

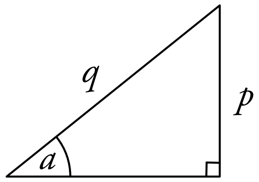
Finding Trigonometric Ratios

You should already be familiar with the following formulae (SOH CAH TOA).



$$\sin a = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \cos a = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \tan a = \frac{\text{Opposite}}{\text{Adjacent}}.$$

If we have $\sin a = \frac{p}{q}$ where $0 < a < \frac{\pi}{2}$, then we can form a right-angled triangle to represent this ratio.



Since $\sin a = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{p}{q}$ then:

- the side opposite a has length p;
- the hypotenuse has length q.

The length of the unknown side can be found using Pythagoras's Theorem.

Once the length of each side is known, we can find $\cos a$ and $\tan a$ using SOH CAH TOA.

The method is similar if we know $\cos a$ and want to find $\sin a$ or $\tan a$.

EXAMPLES

4. Acute angles p and q are such that $\sin p = \frac{4}{5}$ and $\sin q = \frac{5}{13}$. Show that $\sin(p+q) = \frac{63}{65}$.

Confirming Identities

EXAMPLES

5. Show that $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$.

6. Show that $\dfrac{\sin(s+t)}{\cos s \cos t} = \tan s + \tan t$ for $\cos s \neq 0$ and $\cos t \neq 0$.

4 Double-Angle Formulae

Using the compound angle identities with $A = B$, we obtain expressions for $\sin 2A$ and $\cos 2A$. These are called **double-angle formulae**.

$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A.$$

Note that these are given in the exam.

EXAMPLES

1. Given that $\tan\theta = \frac{4}{3}$, where $0 < \theta < \frac{\pi}{2}$, find the exact value of $\sin 2\theta$ and $\cos 2\theta$.

2. Given that $\cos 2x = \frac{5}{13}$, where $0 < x < \pi$, find the exact values of $\sin x$ and $\cos x$.

5 Further Trigonometric Equations

We will now consider trigonometric equations where double-angle formulae can be used to find solutions. These equations will involve:

- $\sin 2x$ and either $\sin x$ or $\cos x$;
- $\cos 2x$ and $\cos x$;
- $\cos 2x$ and $\sin x$.

Remember
The double-angle formulae are given in the exam.

Solving equations involving sin2*x* and either sin*x* or cos*x*

EXAMPLE

1. Solve $\sin 2x° = -\sin x°$ for $0 \leq x < 360$.

Solving equations involving cos2*x* and cos*x*

EXAMPLE

2. Solve $\cos 2x = \cos x$ for $0 \leq x \leq 2\pi$.

Solving equations involving cos2*x* and sin*x*

EXAMPLE

3. Solve $\cos 2x = \sin x$ for $0 < x < 2\pi$.