Higher
Mathematics

UNIT 3 OUTCOME 4

Wave Functions

Contents

HSN23400

OUTCOME 4

Wave Functions

1 Expressing *pcosx + qsinx* in the form *kcos(x – a)*

An expression of the form $p\cos x + q\sin x$ can be written in the form $k\cos(x-a)$ where

$$k = \sqrt{p^2 + q^2} \quad \text{and} \quad \tan a = \frac{k\sin a}{k\cos a}.$$

The following example shows how to achieve this.

EXAMPLES

1. Write $5\cos x° + 12\sin x°$ in the form $k\cos(x° - a°)$ where $0 \le a \le 360$.

Step 1
Expand $k\cos(x-a)$ using the compound angle formula.

Step 2
Rearrange to compare with $p\cos x + q\sin x$.

Step 3
Compare the coefficients of $\cos x$ and $\sin x$ with $p\cos x + q\sin x$.

Step 4
Mark the quadrants on a CAST diagram, according to the signs of $k\cos a$ and $k\sin a$.

Step 5
Find k and a using the formulae above (a lies in the quadrant marked twice in *Step 4*).

Step 6
State $p\cos x + q\sin x$ in the form $k\cos(x-a)$ using these values.

2. Write $5\cos x - 3\sin x$ in the form $k\cos(x-a)$ where $0 \le a \le 2\pi$.

Note
Make sure your calculator is in radian mode.

2 Expressing *p*cos*x* + *q*sin*x* in other forms

An expression in the form $p\cos x + q\sin x$ can also be written in any of the following forms using a similar method:

$$k\cos(x+a), \qquad k\sin(x-a), \qquad k\sin(x+a).$$

EXAMPLES

1. Write $4\cos x° + 3\sin x°$ in the form $k\sin(x° + a°)$ where $0 \le a \le 360$.

2. Write $\cos x - \sqrt{3} \sin x$ in the form $k\cos(x+a)$ where $0 \le a \le 2\pi$.

3   Multiple Angles

We can use the same method with expressions involving the same multiple angle, i.e. $p\cos(nx) + q\sin(nx)$, where n is a constant.
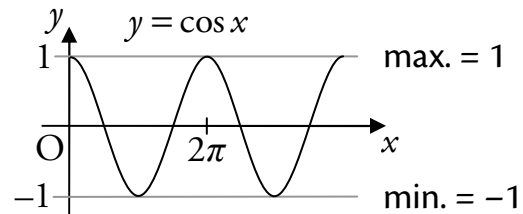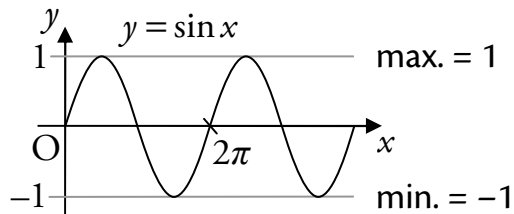
EXAMPLE

Write $5\cos 2x° + 12\sin 2x°$ in the form $k\sin(2x° + a°)$ where $0 \le a \le 360$.

4   Maximum and Minimum Values

To work out the maximum or minimum values of $p\cos x + q\sin x$, we can rewrite it as a single trigonometric function, e.g. $k\cos(x-a)$.

Recall that the maximum value of the sine and cosine functions is 1, and their minimum is -1.

EXAMPLE

Write $4\sin x + \cos x$ in the form $k\cos(x-a)$ where $0 \le a \le 2\pi$ and state:

(i) the maximum value and the value of $0 \le x < 2\pi$ at which it occurs

(ii) the minimum value and the value of $0 \le x < 2\pi$ at which it occurs.

5   Solving Equations

The method of writing two trigonometric terms as one can be used to help solve equations involving both a $\sin(nx)$ and a $\cos(nx)$ term.

EXAMPLES

1. Solve $5\cos x° + \sin x° = 2$ where $0 \le x \le 360$.

2. Solve $2\cos 2x + 3\sin 2x = 1$ where $0 \le x \le 2\pi$.

2. Solve $2\cos 2x + 3\sin 2x = 1$ where $0 \le x \le 2\pi$.

6   Sketching Graphs of *y* = *p*cos*x* + *q*sin*x*

Expressing $p\cos x + q\sin x$ in the form $k\cos(x-a)$ enables us to sketch the graph of $y = p\cos x + q\sin x$.

EXAMPLES

1. (a) Write $7\cos x° + 6\sin x°$ in the form $k\cos(x° - a°)$, $0 \le a \le 360$.

   (b) Hence sketch the graph of $y = 7\cos x° + 6\sin x°$ for $0 \le x \le 360$.

2. Sketch the graph of $y = \sin x° + \sqrt{3} \cos x°$ for $0 \leq x \leq 360$.

3. (a) Write $5\sin x° - \sqrt{11}\cos x°$ in the form $k\sin(x° - a°)$, $0 \le a \le 360$.

   (b) Hence sketch the graph of $y = 5\sin x° - \sqrt{11}\cos x° + 2$, $0 \le x \le 360$.