



DUNBAR GRAMMAR SCHOOL

Higher Mathematics Revision for final exam



To revise the topics indicated, attempt the selected questions from the exam papers, check answers and method and ask your teacher to give help when required.

"The best way to eat an elephant is one bite at a time. The secret is not to waste too much time deciding where to take the first bite."

Unknown

"The secret of getting ahead is getting started. The secret of getting started is breaking your complex, overwhelming tasks into small manageable tasks, and then starting with the first one."

Mark Twain

Completed by	Topic	
13 weeks to go	Straight Line	Differentiation
11 weeks to go	Sequences	Vectors
9 weeks to go	Functions	Quadratics
8 weeks to go	Trig Equations	Addition and Double Angle Formulae
7 weeks to go	Integration & Areas	Trig. Graphs
6 weeks to go	Inc. and Dec. Functions and Turning Points	Circles
5 weeks to go	Polynomials and Remainder Theorem	Wave Function
4 weeks to go	Logs & Exponentials	2011 Paper
2 week to go	2016 Paper 1	2016 Paper 2
Exam week	2017 Paper 1	2017 Paper 2

About the revision plan

The aim of this revision guide is to help you plan out your revision and shows just how much there is to revise for. Can you imagine trying to revise all of this the week before the exam?!! It pays dividends to begin revising early.

Attempt the tasks each week and come to the after school revision sessions to get help with questions you found difficult.

This revision plan uses four exam papers 2009 to 2013 to help revise the topics in this course. We have also saved the 2016 & 2017 papers as full papers to practice at the end of the course. All the past papers, new higher papers and solutions are available at <https://sites.google.com/a/edubuzz.org/maths-past-papers/home/>

☺☺ Happy Revising ☺☺

Exam Technique

Often the biggest problem for students at higher level is understanding how to start a question. In the table below there are some clues as to the things to look out for, and the first step to take in common exam questions.

Topic	Key Phrases	First Step
Straight Line	“Find the equation of the line...” “Find the equation of the median...”	Identify gradient and a point on the line.
	“Find the point of intersection...”	Set two equations equal to each other.
Composite Functions	“ $f(g(x))$ ”	Sub function into brackets.
Graphs of Functions	“ $kf(x)$ ”, “ $f(x - a)$ ”, etc.	Decide how variables will affect the graph.
Differentiation	“tangent to the curve”, “ $f'(x)$ ”, “ $\frac{dy}{dx}$ ”	Arrange equation in index form.
	“stationary point”	Differentiate and set equal to zero.
	“maximise”, “minimise”	Find a stationary point.
Recurrence Relations	“ U_n ”, “Limit”	Set up recurrence relation
Synthetic Division	“ $x^3 \dots$ ”	Arrange the nested calculation
Integration	“Find the area...” or shaded area on graph.	Set up integral with limits
	“Find y in terms of x ” or “Find $f(x)$ ”	Integrate $\frac{dy}{dx}$ or $f'(x)$
Trigonometry	Presence of $\sin(a + b)$, $\cos(a + b)$, $\sin(2x)$ or $\cos(2x)$.	Use formulae sheet to make a substitution.
Circles	Diagram involving a circle or an equation with x^2 and y^2 .	
	“find the equation of the circle”	Locate centre and find radius.
	“find the equation of the tangent”	Find the gradient of the radius
	“find the intersection” between a circle and a line or curve.	Sub eqn for the line or curve into the equation of your circle to eliminate y or x .
Vectors	“Find the angle between...”	$\cos \theta = \frac{a \cdot b}{ a b }$
	“perpendicular”	Show that $a \cdot b = 0$
Logarithms	“log”	Use log rules to simplify to a single expression on each side of equation.
Exponentials	Exponential function given in question.	Varies
Wave Function	“ $k \sin(x \pm a)$ ” or “ $k \cos(x \pm a)$ ”	Expand formula using formula sheet

Polynomials and Remainder Theorem

You need to know:

- (a) that the remainder on dividing a polynomial $f(x)$ by $(x - h)$ is $f(h)$
- (b) that if the remainder on dividing a polynomial $f(x)$ by $(x - h)$ is zero then $(x - h)$ is a factor of $f(x)$ and conversely.
- (c) how to determine the factors of a polynomial.
- (d) how to determine the roots of a polynomial equation

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)		16, 18, 22	7, 17	3, 21a	6, 21
Paper 2 (Calculator)	3a	6b	2		3a

Differentiation

You need to know:

- (a) the notations $f'(x)$ and $\frac{dy}{dx}$ for a derivative.
- (b) if $f(x) = x^n$ then $f'(x) = nx^{n-1}$
- (c) if $f(x) = g(x) + h(x)$ then $f'(x) = g'(x) + h'(x)$
- (d) if $f(x) = kg(x)$ where k is a constant then $f'(x) = kg'(x)$
- (e) the meaning of the terms: rate of change and average gradient
- (f) $f'(a)$ is the rate of change of f at a
- (g) how to find the gradient of the tangent to a curve and the equation of the tangent
- (h) how to find the points on a curve at which the gradient has a given value
- (i) how to differentiate with respect to x and with respect to other letters
- (j) how to sketch $y = f'(x)$ given the graph of $y = f(x)$
- (k) that if $f(x) = g(h(x))$ then $f'(x) = g'(h(x)) \cdot h'(x)$

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	18	12, 17	4	2, 6, 8	2, 18
Paper 2 (Calculator)	1, 2b*	6a			3b, 7

*2009 Q2b, answer to 2a: $p(x) = 3x^2 - 5$ & $q(x) = 9x^2 + 6x + 1$

"Work joyfully and peacefully knowing the right thoughts and right efforts inevitably bring the right results"

James Allen

Sequences and Recurrence Relations

You need to know:

- (a) the meaning of the terms: sequence, n^{th} term, limit as n tends to infinity
- (b) to use u_n for the n^{th} term of a sequence
- (c) how to use recurrence relations of the form $u_{n+1} = mu_n + c$
- (d) how to calculate a limit

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	1, 6	2, 7		1	8
Paper 2 (Calculator)			3	6	1

Straight Line

You need to know:

- (a) the gradient of a straight line is the tangent of the angle made by the line and the positive direction of the x -axis ($m = \tan\theta$)
- (b) the gradient if the line is determined by the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$ provided $x_1 \neq x_2$
- (c) the equation of a straight line is of the form $Ax + By + C = 0$ and conversely.
- (d) the equation of a straight line is of the form $y - b = m(x - a)$
- (e) the equation of a straight line is of the form $y = mx + c$
- (f) the gradients of parallel lines are equal
- (g) the lines with gradients m_1 and m_2 are parallel if and only if $m_1 \times m_2 = -1$
- (h) an altitude of a triangle is a line from a vertex perpendicular to the opposite side
- (i) a median of a triangle is a line from a vertex to the midpoint of the opposite side
- (j) how to find the coordinates of the midpoint of two given points
- (k) the perpendicular bisector of a line AB is a line through the midpoint of AB and perpendicular to AB.
- (l) how to find the intersection of two lines
- (m) how to find the gradient of a straight line if you know its equation ($y = mx + c$)
- (n) how to find the distance between two given points

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	3, 5, 15, 21	1, 21, 23a(i)	2, 8, 21	4, 23	5
Paper 2 (Calculator)					2

Addition and Double Angle Formulae

You need to know:

- (a) π radians = 180°
- (b) the exact values of sin, cos and tan of $\pi/3, \pi/4, \pi/6$ etc.

<ul style="list-style-type: none"> (c) the formulae $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ (d) the formulae $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ (e) the formulae $\sin 2x = 2 \sin x \cos x$ (f) the formulae $\cos 2x = \cos^2 x - \sin^2 x$ $= 2 \cos^2 x - 1$ $= 1 - 2 \sin^2 x$ 	}	On formulae sheet in exam
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- (f) how to solve equations involving the above formulae
- (g) how to interpret and draw graphs involving the above formulae

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	7, 11, 24	23	10, 12, 23	5	9, 10
Paper 2 (Calculator)	5	4			8

"Lucky? Lucky? The funny thing is - the more I practice the luckier I get."

Gary Player
British Open, US Open and US Masters champion

After American journalists described his bunker shot which won the US Open as lucky

"Never make the mistake of assuming that people who are successful have just been lucky."

Unknown

Increasing, Decreasing Functions and Turning Points

You need to know:

- (a) the meaning of the terms: increasing, decreasing, stationary point, stationary value.
- (b) how to determine whether a function f is increasing, decreasing or stationary at $x = a$
- (c) the meaning of the terms: maximum turning point, minimum turning point, horizontal point of inflexion.
- (d) how to determine the stationary points of $y = f(x)$
- (e) how to determine the nature of stationary points
- (f) how to determine the greatest and least value of a function on a given interval.
- (g) how to sketch the curve of $y = f(x)$

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)		15	18, 22a,b	18	
Paper 2 (Calculator)	1	5		3	

Trig Graphs

You need to know:

- (a) the meaning of the terms: amplitude and period
- (b) how to draw the graphs of $f(x) = A\sin Bx + C$ and $f(x) = A\cos Bx + C$ for suitable constants A, B, C .
- (c) how to interpret the above graphs

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calculator)		4		9, 12	4
Paper 2 (Calculator)	5a				

"Many of life's failures are people who did not realise how close they were to success when they gave up."

Thomas Edison

Exponential Functions and Logarithms

You need to know:

- (a) the general features of the graph of $f(x) = a^x$ ($a > 1$ and $0 < a < 1$, $a \in \mathbb{R}$)
- (b) the general features of the graph of $f(x) = \log_a x$ ($a > 1$, $x > 0$)
- (c) $a^y = x \leftrightarrow \log_a x = y$ ($a > 1$, $x > 0$)
- (d) how to apply the laws of logarithms and indices
- (e) how to recognise the probable form of a function from its graph when the probable form is logarithmic or exponential
- (f) how to use a straight line graph to confirm a relationship of the form $y = ax^b$ or $y = ab^x$
- (g) how to solve for a and b equations of the form
 - $\log y = a \log x + b$
 - $y = a x^b$
 - $y = a b^x$
 given two pairs of corresponding values of x and y

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	6	19	11, 16	20	20
Paper 2 (Calculator)	3b*	7		7	5

* Answer to 2009 Q3a): $x^3 + 8x^2 + 11x - 20 = (x - 1)(x + 4)(x + 5)$

Integration

You need to know:

- (a) the meaning of the terms: integral, integrate, constant of integration, definite integral, limits of integration, indefinite integral
- (b) how to find the integrals of functions $f(x) = px^n$ for all except $n = -1$) and the sum or difference of such functions
- (c) if $f(x) = F'(x)$ then $\int f(x) dx = F(x) + c$
- (d) if $f(x) = F'(x)$ then $\int_a^b f(x) dx = F(b) - F(a)$
- (e) how to solve equations of the form $\frac{dy}{dx} = f(x)$
- (f) how to evaluate definite integrals

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)			11, 16	11	7
Paper 2 (Calculator)					

Quadratic Functions and Equations

You need to know:

- (a) the roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- (b) the discriminant is $b^2 - 4ac$
- (c) how to determine whether or not the roots of a quadratic are real and, if real, whether equal or unequal (distinct)
- (d) how to complete the square in $ax^2 + bx + c$
- (e) how to solve a quadratic inequation
- (f) how to determine a quadratic equation with given roots

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	12, 19	5, 6, 13, 18	5, 9	13, 19	3, 19, 21
Paper 2 (Calculator)				1b	

Wave Function (Acosx + Bsinx)

You need to know:

- (a) how to find k and α such that $k > 0$ and $k\cos(x - \alpha) = A\cos x + B\sin x$
- (b) how to find the maximum / minimum values of $A\cos x + B\sin x$ and the corresponding values of x .
- (c) how to solve equations of the form $A\cos x + B\sin x = C$
- (d) how to determine the solution of trig equations such as $3\cos(2x - \pi/2) = 1, 0 \leq x \leq \pi$

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	13			22	23
Paper 2 (Calculator)		2	6a		

"The biggest human temptation is to settle for too little."

Thomas Merton

Circle

You need to know:

- (a) the equation of a circle (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$
- (b) the equation $x^2 + y^2 + 2gx - 2fy + c = 0$ represents a circle with centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$ provided $g^2 + f^2 - c > 0$
- (c) how to find the equation of a circle
- (d) how to find the points at which a given line intersects a circle
- (e) how to determine whether a given line is a tangent to a given circle
- (f) how to determine whether two given circles touch each other
- (g) how to find the equation of the tangent at a given point on a given circle

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	2, 9	8	6		22
Paper 2 (Calculator)	4	3	7	2	

Areas

You need to know:

- (a) the meaning of the term "area under a curve"
- (b) the area bounded by the curve $y = f(x)$, the axis and the lines $x = a$ and $x = b$ is

$$\int_a^b f(x) dx \text{ if } f(x) \geq 0 \text{ for } a \leq x \leq b$$

$$\text{and is } -\int_a^b f(x) dx \text{ if } f(x) \leq 0 \text{ for } a \leq x \leq b$$

- (c) how to determine the area bounded by the curve $y = f(x)$ and the x -axis and the lines $x = a$ and $x = b$
- (d) if $f(x) \geq g(x)$ for $a \leq x \leq b$ then the area enclosed by the curves $y = f(x)$ and $y = g(x)$ and the lines $x = a$ and $x = b$ is

$$\int_a^b (f(x) - g(x)) dx$$

- (e) how to determine the area bounded by two curves

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	16	14		21b	
Paper 2 (Calculator)		6c	4		4

Functions

You need to know:

- (a) the meaning of the terms: domain, codomain, range, inverse of a function, composite function
- (b) the notation $f(g(x))$ for a composite function
- (c) the notation $f^{-1}(x)$ for the inverse of function $f(x)$
- (d) how to sketch, if you are given the graph of $f(x)$, the graphs of related functions such as:
 - $-f(x)$
 - $f(x) + a$
 - $f(x + b)$
 - $kf(x)$

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	10, 14, 23	11, 20	3, 20, 22c		1, 11, 13, 15, 17
Paper 2 (Calculator)	2a			1a, 4	

Vectors

You need to know:

- (a) the meaning of the terms: vector, magnitude, addition, scalar multiples, position vector, unit vector, directed line segment, component, distance between points, scalar product
- (b) that two vectors are equal if their corresponding components are equal and conversely
- (c) that if A, P and B are collinear points such that $\frac{AP}{PB} = \frac{m}{n}$
 then $\vec{AP} = \frac{m}{n} \vec{PB}$
- (d) that \mathbf{i} , \mathbf{j} and \mathbf{k} can be used as basis vectors
- (e) that if \mathbf{u} and \mathbf{v} are vectors that can be represented by parallel lines then $\mathbf{u} = k\mathbf{v}$ where k is a constant and conversely
- (f) that the scalar product $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ and is also equal to $a_1b_1 + a_2b_2 + a_3b_3$
- (g) how to evaluate the scalar product
- (h) that $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
- (i) how to find the angle between two directed line segments using the scalar product
- (j) how to determine whether three points with given coordinates are collinear
- (k) how to determine the coordinates or position vector of the point which divides the join of two given points in a given numerical ratio
- (l) how to determine whether or not two vectors, given in component form, are perpendicular

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	17, 22	3, 10, 21c	1, 14	7, 10, 15, 17	12, 14, 24
Paper 2 (Calculator)	7	1	1	5	

Further Differentiation and Integration (incl. Trig)

You need to know:

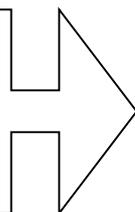
(a) that if $f(x) = g(h(x))$ then $f'(x) = g'(h(x)) \cdot h'(x)$

(b) that if $f(x) = \sin x$ then $f'(x) = \cos x$

(c) that if $f(x) = \cos x$ then $f'(x) = -\sin x$

(d) how to integrate $p \cos qx$

(e) how to integrate $p \sin qx$



On
formulae
sheet in
exam

(f) how to integrate $(px + q)^n$ for all n except $n = -1$

(g) how to integrate sums or differences of functions given in (d), (e), (f)

Questions to attempt:

	2009	2010	2011	2012	2013
Paper 1 (Non-Calc)	18	9		14, 16	16, 18
Paper 2 (Calculator)	5c*				6, 9

*2009 Q5 previous answers: $g(x) = 3\cos(2x)$. Points of intersection are at $x = 0.6$ and $x = 2.6$

After school revision!

Come along and get help with any aspect of the course that is bothering you, or just take the opportunity to revise with a teacher on hand to help clear up any problems.

BOOK RETURN

Please return Maths text books, study packs and any other material that you have borrowed **on the day of the Maths exam**. A box for book return will be outside the exam hall. Ensure that you fill in your book return form.