Higher Physics

Unit 2  Particles and Waves

Section 7   Spectra
Section 7 Spectra

Note Making
Make a dictionary with the meanings of any new words.

Irradiance and the inverse square law
1. Write down the definition of irradiance, the formula and briefly state why it is important.
2. Describe an experiment to measure the relationship between distance and irradiance and how an inverse square law is produced.
3. Briefly describe how we can measure the distance to the moon.

The Bohr model of the atom
1. State Bohr’s proposals for his new model of the atom.
2. Copy the energy level diagram on page 12 and describe what happens when an electron makes a transition from a higher energy state to a lower energy state. Copy the transitions on to your energy level diagram.
3. Do question 4 on page 17 as an example.
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<td>a) Irradiance and the inverse square law.</td>
<td>Investigating irradiance as a function of distance from a point light source. Irradiance as power per Unit area.</td>
<td>Galactic distances and Hubble’s Law. Application to other e-m radiation (eg gamma radiation) Comparing the irradiance from a point light source with a laser.</td>
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<td>b) Line and continuous emission spectra, absorption spectra and energy level transitions.</td>
<td>The Bohr model of the atom. Electrons can be excited to higher energy levels by an input of energy. Ionisation level is the level at which an electron is free from the atom. Zero potential energy is defined as equal to that of the ionisation level, implying that other energy levels have negative values. The lowest energy level is the ground state. A photon is emitted when an electron moves to a lower energy level and its frequency depends on the difference in energy levels. Planck’s constant is the constant of proportionality. Absorption lines in the spectrum of sunlight as evidence for the composition of the Sun’s upper atmosphere.</td>
<td>Line and continuous spectra, eg from tungsten filament lamp, electric heater element, fluorescent tube, burning a salt in a Bunsen flame. Discharge lighting, laboratory and extraterrestrial spectroscopy, the standard of time. Lasers.</td>
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Irradiance and the inverse square law

What is irradiance?

Irradiance of radiation is a measure of the radiation falling on a surface. It is defined as the energy falling on a surface per unit time (i.e. the power per unit area). This relationship can be summarised:

\[ I = \frac{P}{A} \]

- \( I \) = irradiance in \( \text{W m}^{-2} \)
- \( P \) = power in watts
- \( A \) = area in \( \text{m}^2 \)

Why does irradiance matter?

An understanding of irradiance is relevant to a range of applications. For example, NASA monitors solar irradiance to understand the activity of the Sun and climate scientists study solar irradiance to research the impact of solar activity on the Earth’s climate. Interactions between solar radiation and the atmosphere of the Earth can impact on air quality, and understanding of irradiance can allow investigation of the composition of the Earth’s atmosphere. Excessive exposure to sunlight has been linked to the development of a range of skin cancers. The performance of solar cells, an increasingly common use of solar radiation as an energy resource, requires an understanding of irradiance.
Investigating irradiance

The relationship between irradiance of a point source and the distance from that source can be investigated using a simple experimental set up: a light source and a light meter.

The graph of a typical set of results from such an experiment is shown below:

It is clear from this graph that the relationship between irradiance and distance is not a linear one.

Graphing average irradiance against $1/d$ ($d =$ distance) demonstrates that average irradiance is not proportional to $1/d$. 
The graph of average irradiance against $1/d^2$ demonstrates a linear relationship.
From the graph:

\[ I \propto \frac{1}{d^2} \]

\[ Id^2 = \text{constant} \]

\[ I_1d_1^2 = I_2d_2^2 \]

\[ I = \text{irradiance in W m}^{-2} \]
\[ d = \text{the distance from a point source in m} \]

This is described as an inverse square law.

A point source is one which irradiates equally in all directions, i.e. the volume that will be irradiated will be a sphere. The surface area of a sphere can be calculated using \[ A = 4\pi r^2 \], i.e. the area which will be irradiated is proportional to \( r^2 \) (or \( d^2 \)).

**Calculating solar irradiance and the power of the Sun: measuring a sunbeam**

Watch the clip from ‘Measuring a sunbeam’ (BBC Wonders of the Solar System episode ‘Empire of the Sun’).

Professor Brian Cox uses a simple technique to measure the solar energy falling on the Earth, recreating an experiment first carried out by Sir John Herschel in 1838. Pause the clip and use your understanding of irradiance to make the calculations on the Sun’s irradiance.

http://www.bbc.co.uk/programmes/p006szxm.
Irradiance and the distance to the moon

When Neil Armstrong and Buzz Aldrin walked on the surface of the moon on 21 July 1969 as part of the Apollo mission, they set up a lunar laser ranging reflector array, a 0.6 m² panel with 100 mirrors. This is the only experiment from the successful Apollo 11 mission that is still running.

![The Apollo 11 lunar laser-ranging retro reflector array © NASA.](image)

A laser pulse is transmitted from a telescope on Earth. The reflectors in the array are designed to send the pulse back precisely to the location from which it was transmitted. Telescopes on Earth receive the returned pulse.

For more than three decades it was possible to use this technique to make observations of the moon’s orbit, the universal gravitational constant, the composition of the moon, and whether Einstein’s theory of relativity works in practice.

Measuring the time for the pulse to be returned allows the moon’s distance to be calculated very precisely. Until 2005 the measurements allowed calculation to within a few centimetres, a very small uncertainty for a distance of approximately 385 000 000 m! The Apache Point Observatory Lunar Laser-ranging Operation (APOLLO for
short) in New Mexico, which came online in 2005, allows these measurements to be made to within a few millimetres.

Among its aims, the APOLLO project will measure the $1/r^2$ law (i.e. the inverse square law) at the lunar distance scale, i.e. approximately $10^{12}$ m.

What makes a laser suitable for these types of experiment over astronomical distances? A laser has a very small beam divergence over a distance, compared with a light source such as an ordinary filament lamp. Nevertheless, the laser beam is approximately 7 km in diameter by the time it reaches the moon.

**Irradiance and the electromagnetic spectrum**

The inverse square law applies to all electromagnetic radiation, i.e. to visible light and all wavelengths on the electromagnetic spectrum, from radio waves to gamma rays.
A model of the atom and the electromagnetic spectrum

Vibrating charges produce radiation. This is the source of electromagnetic radiation, e.g. visible light, X-rays and gamma rays.

At the end of the 19th century, physicists knew that atoms contained electrons and the motion of these electrons resulted in electromagnetic radiation. However, a mystery remained. Solving this mystery, and the work of scientists including Fraunhofer, Brewster, Kirchoff, Bunsen and Ångstrom, took forward a new area of science – spectroscopy – which revolutionised solar physics and astronomy.

Prior to Isaac Newton’s prism experiment in 1666, the nature of celestial bodies such as the Sun and stars was not well understood. The luminescence from such bodies was attributed to an ‘essence’.

In 1802 William Wallaston observed lines in the spectrum from the Sun. The systematic study of these lines was undertaken by a German optician, Joseph Fraunhofer.

© University Corporation for Atmospheric Research
This is a reproduction of Fraunhofer's original 1817 drawing of the solar spectrum. The more prominent dark lines are labelled alphabetically; some of this nomenclature has survived to this day. In the 1850s Kirchoff (a German physicist, 1824–1887) and Bunsen (a German chemist, 1811–1899) took another step forward in spectroscopy. It was discovered that when various substances were held in the flame of a Bunsen burner and the light produced passed through a prism, distinct spectra were seen, which correspond to distinct elements in the substance. You may have carried out similar experiments in science at school, for example burning a copper salt in a Bunsen flame and observing the distinctive green colour, or sodium (yellow flame), or potassium (lilac flame), or calcium (red flame).

In the 1860s this technique was used by Kirchoff and Bunsen to discover two new elements, caesium and rubidium. Other elements that were discovered using this approach were gallium, helium, argon, neon, krypton and xenon.

We can use a spectroscope to observe various light sources around you, e.g. daylight (do not point the spectroscope directly at the Sun) and the artificial lighting within the classroom. If you do not have a spectroscope in school, it is possible to build a simple one that works just as effectively (http://www.exploratorium.edu/spectroscope/).

A spectrophotometer allows you to observe and record data of the spectrum for a range of light sources. You can compare the spectrum of an ordinary filament lamp with time from its first switch-on. Compare this with the spectrum of an ordinary filament lamp with differing voltage across the lamp. Also contrast these with the spectrum of an energy-saving lamp with time.
Max Planck and the quantisation of energy

In 1900 the German physicist Max Planck (1858–1947; Nobel Prize winner 1918) proposed the quantisation of the energy of a vibrating molecule, i.e. it could only take on certain fixed values.

This helped to explain the gaps and lines in the spectra of elements.

He proposed that:

\[ E \propto f \]

with a constant of proportionality \(6.626 \times 10^{-34}\) J s, this came to be known as Planck’s constant (\(h\)).

\[ E = hf \]

\(E =\) energy in joules (J)

\(f =\) the frequency in hertz (Hz)

\(h =\) Planck’s constant = \(6.626 \times 10^{-34}\) J s

This was a revolutionary idea which provided the basis for quantum physics. Einstein built on Planck’s work, proposing that light was also quantised in photons.

The Bohr model of the atom

In 1911–1913 the Danish physicist Neils Bohr (1885–1962; Nobel Prize winner 1922) collaborated in Cambridge and Manchester with JJ Thomson and Ernst Rutherford.

Bohr knew that a charged body in motion must emit energy as electromagnetic radiation.
This meant that orbiting electrons should emit energy, as a result lose energy and spiral into the nucleus of the atom. This would suggest that the atom cannot be stable.

Bohr struggled to reconcile the accepted model of the atom with this conclusion.

In 1913, Bohr proposed a new model of the atom, which was recognised with the award of the Nobel prize for physics in 1922, and (with some further improvement to take account of Heisenberg’s work in the 1920s), remains the model that accounts for the physical and chemical properties of elements today.

In the Bohr model:

- Electrons exist only in allowed orbits. An electron in an allowed orbit does not radiate energy, i.e. Bohr’s model proposed that the classical electromagnetic theory was incorrect. Electrons can jump between allowed states, and this jump is associated with absorption or emission of electromagnetic radiation.
- Electrons in different orbits have different energies, thus these orbits can be considered to be energy levels.
- Electrons tend to occupy the lowest available energy level, i.e. the level closest to the nucleus.

The allowed orbits of electrons can be represented in an energy level diagram. The diagram below represents the energy levels within a hydrogen atom.

Electrons can exist either in the ground state $E_0$ or in various excited states $E_1$–$E_5$. 
An electron in an excited state can make a transition into a lower energy state.

In a hydrogen atom, the possible transitions are:

E1 → E0, E2 → E0, E3 → E0, E4 → E0, E5 → E0
E2 → E1, E3 → E1, E4 → E1, E5 → E1
E3 → E2, E4 → E2, E5 → E2
E4 → E3, E5 → E3
E5 → E4

When an electron makes a transition from a higher energy state to a lower energy state where does the excess energy go? It is emitted as a photon of electromagnetic radiation. In the Bohr model, it follows that the possible excess energies must be quantised, i.e. in specific ‘packets’, since the electron can make transitions only between specified energy levels.
The frequency of the emitted radiation is proportional to the difference in the energy levels ($\Delta E$):

$$\Delta E \propto f$$

It follows that since $\Delta E$ is quantised, the photons emitted can only be of specified frequency.

**The Bohr model and spectra**

This theory applies only to free atoms. A free atom is one that is not affected by neighbouring atoms, as in a gas. In sources such as filament lamps, the atoms are not free and their electrons may be shared in bonding with other atoms. This results in an infinite number of possible transitions, giving an infinite number of lines, i.e. a continuous spectrum.
Spectra Problems

Irradiance and inverse square law

1. A satellite is orbiting the Earth. The satellite has solar panels, with a total area of 15 m², directed at the Sun. The Sun produces an irradiance of 1·4 kW m⁻² on the solar panels. Calculate the power received by the solar panels.

2. A 100 W light source produces an irradiance of 0·2 W m⁻² at a distance of 2 m.

The light source can be considered to be a point source.

Calculate the irradiance produced at a distance of:

a) 1 m from the source
b) 4 m from the source.

3. An experiment is performed to measure the irradiance produced at different distances from a point source of light. The results obtained are shown in the table.

<table>
<thead>
<tr>
<th>Distance from point source d /m</th>
<th>1·0</th>
<th>1·4</th>
<th>2·2</th>
<th>2·8</th>
<th>3·0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irradiance I /W m⁻²</td>
<td>85</td>
<td>43</td>
<td>17·6</td>
<td>10·8</td>
<td>9·4</td>
</tr>
</tbody>
</table>

a) Sketch the apparatus that could be used to obtain these results.

b) Use an appropriate format to show the relationship between the irradiance I and the distance d.

c) Calculate the irradiance at a distance of 5 m from the source.

d) At what distance from the source is the irradiance 150 W m⁻²?
4. The radiation from the Sun produces an irradiance of 200 W m\(^{-2}\) at a certain point on the surface of the Earth.

   a) What area of solar cells would be required to produce a power output of 1 MW when the cells are considered to be 100% efficient?

   b) The cells are only 15% efficient. What additional area of solar cells is required to produce a power output of 1 MW?

5. An experiment is set up in a darkened laboratory with a small lamp L1 with a power \(P\). The irradiance at a distance of 0·50 m from the lamp is 12 W m\(^{-2}\). The experiment is repeated with a different small lamp L2 that emits a power of 0·5 \(P\).

   Calculate the irradiance at a distance of 0·25 m from this lamp.

**Line and continuous spectra**

1. When the light emitted by a particular material is observed through a spectroscope, it appears as four distinct lines.

   a) What name is given to this kind of emission spectrum?

   b) Explain why a series of specific, coloured lines is observed.

   c) The red line in the spectrum coincides with a wavelength of 680 nm.
Calculate the energy of the photons of light that produced this line.

d) The spectroscope is now used to examine the light emitted from a torch bulb (filament lamp). What difference is observed in the spectrum when compared with the one in the diagram?

2. The diagram shows some of the energy levels for two atoms X and Y.

\[ \begin{align*} 
X_2 & \quad Y_3 \\
X_1 & \quad Y_2 \\
X_0 & \quad \text{ground state} \quad Y_0 
\end{align*} \]

(a) (i) How many downward transitions are possible between these energy levels of each atom?

(ii) How many lines could appear in the emission spectrum of each element as a result of these energy levels?

(iii) Copy the diagram of the energy levels for each atom and show the possible transitions.

(b) Which transition in each of these diagrams gives rise to the emitted radiation of:

(i) lowest frequency
(ii) shortest wavelength?

3. The diagram shows some of the electron energy levels of a particular element.

\[ \begin{align*} 
E_3 & \quad -2.62 \times 10^{-19} \text{ J} \\
E_2 & \quad -4.08 \times 10^{-19} \text{ J} \\
E_1 & \quad -7.63 \times 10^{-19} \text{ J} \\
E_0 & \quad -15.83 \times 10^{-19} \text{ J} 
\end{align*} \]
(a) How many lines could appear in the emission spectrum of this element as a result of these levels?

(b) Calculate the frequencies of the photons arising from:
   (i) the largest energy transition
   (ii) the smallest energy transition.
   (iii) Show whether any of the emission lines in the spectrum correspond to frequencies within the visible spectrum.
   (iv) Explain which transition would produce the photons most likely to cause photoemission in a metal.

4. The diagram shows some of the electron energy levels in a hydrogen atom.

   \[
   \begin{align*}
   W_3 & \quad \text{\underline{\ldots\ldots\ldots}} \quad -1.360 \times 10^{-19} \text{ J} \\
   W_2 & \quad \text{\underline{\ldots\ldots\ldots}} \quad -2.416 \times 10^{-19} \text{ J} \\
   W_1 & \quad \text{\underline{\ldots\ldots\ldots}} \quad -5.424 \times 10^{-19} \text{ J} \\
   W_0 & \quad \text{\underline{\ldots\ldots\ldots}} \quad -21.76 \times 10^{-19} \text{ J}
   \end{align*}
   \]

a) How many emission lines are possible from electron transitions between these energy levels?

b) Which of the following radiations could be absorbed by the electrons in a hydrogen atom?

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2.92 \times 10^{15}) Hz</td>
<td>(4.89 \times 10^{-7}) m.</td>
</tr>
<tr>
<td>(1.57 \times 10^{15}) Hz</td>
<td></td>
</tr>
</tbody>
</table>
5. Explain why the absorption spectrum of an atom has dark lines corresponding to frequencies present in the emission spectrum of the atom.

6. (a) Explain the presence of the Fraunhofer lines, the dark lines that appear in the spectrum of sunlight.

(b) How are Fraunhofer lines used to determine the gases that are present in the solar atmosphere?

7. The light from a star can be analysed to show the presence of different elements in the star. How can the positions of the spectral lines for the elements be used to determine the speed of the star?
8. A bunsen flame is placed between a sodium vapour lamp and a screen as shown.

A sodium ‘pencil’ is put into the flame to produce vaporised sodium in the flame.

a) Explain why a dark shadow of the flame is seen on the screen.

b) The sodium vapour lamp is now replaced with a cadmium vapour lamp. Explain why there is now no dark shadow of the flame on the screen.
Solutions

Irradiance and inverse square law

1. 21 kW
2. (a) 0.8 Wm\(^{-2}\)
   (b) 0.05 Wm\(^{-2}\)
3. (c) 3.4 Wm\(^{-2}\)
   (d) 0.75 m
4. (a) 5000 m\(^2\)
   (b) 28333 m\(^2\)
5. 24 Wm\(^{-2}\)

Line and continuous spectra

1. (c) 2.93 \times 10^{-19} J
2. (a) (i) X 3; Y 6
   (ii) X 3; Y 6
   (b) (i) X\(_2\) to X\(_3\); Y\(_3\) to Y\(_2\)
   (ii) X\(_2\) to X\(_0\); Y\(_3\) to Y\(_0\)
3. (a) 6 lines
   (b) (i) 2.0 \times 10^{15} Hz
   (ii) 2.2 \times 10^{14} Hz
4. (a) 6 lines