SCHOLAR Study Guide

National 5 Mathematics

Course Materials
Topic 22: Surds and indices

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Topic 1

Surds and indices

Contents

Learning objectives

By the end of this topic, you should be able to

- *simplify surds;*
- *rationalise denominators;*
- *multiply and divide using positive, negative and fractional indices;*
- *raise a power to a power and know what a power of zero means.*

1.1 Simplifying surds

Simplifying surds


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A surd is an irrational number, which cannot be worked out exactly. It is a square root, cube root, etc.

For example, $\sqrt{2}$, $\sqrt{7}$, $\sqrt[3]{5}$ are all surds but $\sqrt{25}$, $\sqrt{100}$, $\sqrt[3]{8}$ are not since $\sqrt{25} = 5$ and $\sqrt[3]{8} = 2$.

Work through the following examples and take note of the general rule for multiplying surds.

$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$
$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$
$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$
$\sqrt{50} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{2}$

Key point

Rule for multiplication: $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

. . . . . .

Examples

1.

Problem:

Simplify $\sqrt{8}$

Solution:

First we look for factors of 8 that are **squares**.

8 can be expressed as 4×2.

Using the rule for multiplication gives: $\sqrt{8} = \sqrt{(4 \times 2)} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

. . . . . .

2.

Problem:

Simplify $\sqrt{24}$

Solution:

First we look for factors of 24 that are squares.

24 can be expressed as 4×6.

Using the rule for multiplication gives: $\sqrt{24} = \sqrt{(4 \times 6)} = \sqrt{4}\sqrt{6} = 2\sqrt{6}$

. . . . . .

3.

Problem:

Simplify $\sqrt{48}$

Solution:

First we look for factors of 48 that are squares.

48 can be expressed as 4×12.

Using the rule for multiplication gives: $\sqrt{48} = \sqrt{(4 \times 12)}$

but 12 can also be expressed as 4×3 giving

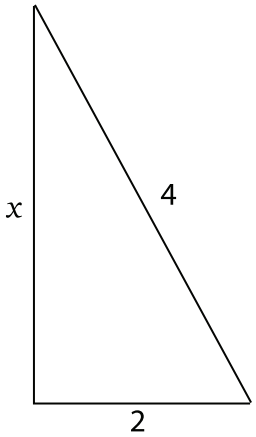
$\sqrt{48} = \sqrt{(4 \times 12)} = \sqrt{(4 \times 4 \times 3)} = \sqrt{4} \times \sqrt{4} \times \sqrt{3} = 2 \times 2 \times \sqrt{3} = 4\sqrt{3}$

. . . . . .

4.

Problem:

Use Pythagoras to calculate x, leaving your answer as a surd.



Solution:

$x = \sqrt{4^2 - 2^2} = \sqrt{16 - 4} = \sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

. . . . . .

Go online

Simplifying surds: Rule for multiplication practice

Q1: Simplify $\sqrt{45}$

. . . . . .

Q2: Simplify $\sqrt{72}$

. . . . . .

Q3: Simplify $\sqrt{32}$

. . . . . .

Q4: Simplify $\sqrt{50}$

. . . . . .

Q5: Simplify $\sqrt{54}$

. . . . . .

Q6: Simplify $\sqrt{80}$

. . . . . .

Key point

Rule for Division

When dividing surds, we follow a similar pattern.

$$\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2} \qquad \sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4} \qquad \sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$$

General Rule

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Examples

1.

Problem:

Simplify $\sqrt{\frac{20}{5}}$

Solution:

$\sqrt{\frac{20}{5}} = \sqrt{4} = 2$

. . . . . .

2.

Problem:

Simplify $\sqrt{\frac{18}{75}}$

Solution:

$\sqrt{\frac{18}{75}} = \frac{\sqrt{18}}{\sqrt{75}} = \frac{\sqrt{9\times 2}}{\sqrt{25\times 3}} = \frac{3\sqrt{2}}{5\sqrt{3}}$

. . . . . .

3.

Problem:

Simplify $\sqrt{\frac{12}{75}}$

Solution:

$\sqrt{\frac{12}{75}} = \frac{\sqrt{12}}{\sqrt{75}} = \frac{\sqrt{4}\times\sqrt{3}}{\sqrt{25}\times\sqrt{3}} = \frac{2\sqrt{3}}{5\sqrt{3}} = \frac{2}{5}$

Note that the $\sqrt{3}$ on the numerator and denominator cancel each other out.

. . . . . .

Go online

Simplifying surds: Rule for division practice

Q7: Simplify $\sqrt{\frac{80}{5}}$

.....

Q8: Simplify $\frac{\sqrt{98}}{\sqrt{28}}$

.....

Q9: Simplify $\frac{\sqrt{32}}{\sqrt{20}}$

.....

Q10: Simplify $\frac{\sqrt{200}}{\sqrt{50}}$

.....

Q11: Simplify $\sqrt{\frac{45}{12}}$

.....

Q12: Simplify $\sqrt{\frac{54}{30}}$

.....

Go online

Simplifying surds exercise

These questions are for practice only.

Q13: Simplify $\sqrt{27}$

.....

Q14: Simplify $\sqrt{80}$

.....

Q15: Simplify $\sqrt{300}$

.....

Q16: Simplify $\sqrt{44}$

.....

Q17: Simplify $\sqrt{98}$

.....

Q18: Simplify $\sqrt{125}$

.....

Q19: Simplify $\sqrt{\frac{16}{9}}$

.....

Q20: Simplify $\sqrt{\frac{24}{49}}$

.....

Q21: Simplify $\sqrt{\frac{81}{125}}$

.....

Q22: Simplify $\sqrt{\frac{32}{18}}$

.....

Q23: Simplify $\sqrt{\frac{75}{72}}$

.....

1.2 Collecting like terms

Surds can also be simplified using the rules you already know from Algebra.

Key point

You know that $3x + 5x = 8x$ so it follows that $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$
The same rules apply to subtraction for example, $5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$
The rules can be combined for example, $2\sqrt{5} + 5\sqrt{5} - \sqrt{5} = 6\sqrt{5}$
You CANNOT simplify $3\sqrt{2} + 5\sqrt{3}$

Collecting like terms practice: Addition and subtraction


Go online

Q24: $3\sqrt{3} + 2\sqrt{3}$

.....

Q25: $\sqrt{2} + 3\sqrt{2} + 7\sqrt{2}$

.....

Q26: $6\sqrt{3} - 2\sqrt{3}$

.....

Q27: $3\sqrt{5} - 2\sqrt{5}$

.....

Q28: $5\sqrt{7} - 2\sqrt{7} + \sqrt{7}$.

.....

Key point

We can also use the rules of simplifying surds then collecting like terms.
For example,

$$\sqrt{50} + 3\sqrt{2} = \sqrt{25} \times \sqrt{2} + 3\sqrt{2}$$
$$= 5\sqrt{2} + 3\sqrt{2}$$
$$= 8\sqrt{2}$$

Combining simplifying surds and collecting like terms practice


Go online

Q29: Simplify $\sqrt{18} + \sqrt{2}$

.....

Q30: Simplify $7\sqrt{3} - \sqrt{27}$

.....

Q31: Simplify $\sqrt{32} - 3\sqrt{2}$

.....

Q32: Simplify $2\sqrt{5} + \sqrt{45} - \sqrt{5}$

.....

Go online

Collecting like terms exercise

These questions are for practice only.

Q33: Simplify $\sqrt{3} + 2\sqrt{3}$

.....

Q34: Simplify $5\sqrt{6} - 2\sqrt{6}$

.....

Q35: Simplify $\sqrt{5} + 7\sqrt{5} - 2\sqrt{5}$

.....

Q36: Simplify $5\sqrt{7} - \sqrt{28}$

.....

Q37: Simplify $\sqrt{98} + 2\sqrt{2} - \sqrt{2}$

.....

1.3 Rationalising denominators

The purpose of rationalising a denominator is to turn the denominator into a whole number. Those of you with a modern scientific calculator will find that if you enter $\frac{1}{\sqrt{2}}$ your calculator will automatically rationalise the denominator giving $\frac{\sqrt{2}}{2}$.

The way to rationalise a surd on the denominator is to multiply both the numerator and denominator by the surd.

Examples

1.

Problem:

Rationalise the denominator $\frac{1}{\sqrt{2}}$

Solution:

$\frac{1}{\sqrt{2}} = \frac{1\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$

Note we multiplied by $\frac{\sqrt{2}}{\sqrt{2}}$. Which simplifies to $\frac{\sqrt{2}}{\sqrt{2}} = 1$.

So multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$ will not change the value of $\frac{1}{\sqrt{2}}$ just its appearance.

. . . . . .

2.

Problem:

Rationalise the denominator and simplify $\frac{5}{\sqrt{10}}$

Solution:

$\frac{5}{\sqrt{10}} = \frac{5\times\sqrt{10}}{\sqrt{10}\times\sqrt{10}} = \frac{5\sqrt{10}}{\sqrt{100}} = \frac{5\sqrt{10}}{10} = \frac{\sqrt{10}}{2}$

Note we multiplied by $\frac{\sqrt{10}}{\sqrt{10}}$.

Remember you can simplify $\frac{5}{10}$ to $\frac{1}{2}$.

. . . . . .

3.

Problem:

Rationalise the denominator and simplify $\frac{2\sqrt{3}-1}{2\sqrt{6}}$

Solution:

$\frac{2\sqrt{3}-1}{2\sqrt{6}} = \frac{(2\sqrt{3}-1)\times\sqrt{6}}{2\sqrt{6}\times\sqrt{6}} = \frac{2\sqrt{18}-\sqrt{6}}{2\sqrt{36}} = \frac{2\times\sqrt{9}\times\sqrt{2}-\sqrt{6}}{2\times6} = \frac{6\sqrt{2}-\sqrt{6}}{12}$

Note we multiplied by $\frac{\sqrt{6}}{\sqrt{6}}$.
Remember to multiply out the brackets.

. . . . . .

4.

Problem:

Rationalise the denominator and simplify $\frac{1}{2+\sqrt{3}}$

Solution:

This is the most difficult type of expression to rationalise and simplify because it has a sum on the denominator.

We need to use a difference of two squares to deal with this type of expression.

Remember $(x+y)(x-y) = x^2 - y^2$

So if we multiply the numerator and denominator by $(2-\sqrt{3})$ we will rationalise the denominator.

Giving $\frac{1}{2+\sqrt{3}} = \frac{1\times(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-\sqrt{3}}{2^2-\sqrt{3}^2} = \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} \quad or \quad 2-\sqrt{3}$

Remember $\sqrt{3}^2 = \sqrt{3}\times\sqrt{3} = \sqrt{9} = 3$

. . . . . .

Rationalise the denominator practice

Go online

Rationalise the denominator and simplify:

Q38: $\frac{1}{\sqrt{6}}$

. . . . . .

Q39: $\frac{3}{2\sqrt{3}}$

. . . . . .

Q40: $\frac{\sqrt{2}+1}{\sqrt{10}}$

. . . . . .

Q41: $\frac{1}{\sqrt{5}-1}$

. . . . . .

Go online

Rationalising denominators exercise

These questions are for practice only.

Q42: Rationalise the denominator $\frac{1}{\sqrt{7}}$

. . . . . .

Q43: Rationalise the denominator and simplify $\frac{1}{\sqrt{13}}$

. . . . . .

Q44: Rationalise the denominator and simplify $\frac{4}{\sqrt{2}}$

. . . . . .

Q45: Rationalise the denominator and simplify $\frac{2}{\sqrt{14}}$

. . . . . .

Q46: Rationalise the denominator and simplify $\frac{5}{2\sqrt{10}}$

. . . . . .

Q47: Rationalise the denominator and simplify $\frac{1+\sqrt{5}}{\sqrt{5}}$

. . . . . .

Q48: Rationalise the denominator and simplify $\frac{1}{\sqrt{3}+1}$

. . . . . .

1.4 Multiplication and division of terms with positive indices

Using the laws of indices

Go online

Multiply an expression by a

$a^1 = a$

$a^2 = a \times a$

$a^3 = a \times a \times a$

$a^4 = a \times a \times a \times a$

$a^5 = a \times a \times a \times a \times a$

$a^6 = a \times a \times a \times a \times a \times a$

$a^7 = a \times a \times a \times a \times a \times a \times a$

$a^8 = a \times a \times a \times a \times a \times a \times a \times a$

$a^9 = a \times a \times a \times a \times a \times a \times a \times a \times a$

$a^{10} = a \times a \times a \times a \times a \times a \times a \times a \times a \times a$

Multiply the expression below by $a \times a$

$a^1 \times a^9 = a \times a \times a \times a \times a \times a \times a \times a \times a \times a$

$= a^{10}$

$= a^{1+9}$

General Rule

$a^n \times a^m = a^{n+m}$

$a^6 \times a^4 = a \times a \times a \times a \times a \times a \times a \times a \times a \times a$

$= a^{10}$

$= a^{6+4}$

Multiply the expression below by $\frac{a}{a}$

$$\frac{a^4}{a^1} = \frac{a \times a \times a \times a}{a}$$

$$= \frac{a \times a \times a}{1}$$

$= a^3$

$= a^{4-1}$

General Rule

$$\frac{a^n}{a^m} = a^{n-m}$$

$$\frac{a^7}{a^4} = \frac{a \times a \times a \times a \times a \times a \times a}{a \times a \times a \times a}$$

$= a^3$

$= a^{7-4}$

. . . . . .

Examples

1.

Problem:

Simplify $x^3 \times x^5 \div x^4$

Solution:

$x^3 \times x^5 \div x^4 \ = \ x^{3+5-4} = x^4$

. . . . . .

2.

Problem:

Simplify $\frac{x^5 \times x^4}{x^2}, \ x \neq 0$

Solution:

$\frac{x^5 \times x^4}{x^2} \ = \ x^{5+4-2} \ = \ x^7$

. . . . . .

3.

Problem:

Simplify $3x^{\frac{1}{3}} \ \times \ 4x^{\frac{2}{3}}$

Solution:

$$\begin{aligned} 3x^{\frac{1}{3}} \ \times \ 4x^{\frac{2}{3}} \ &= \ 3 \ \times \ 4 \ \times \ x^{\frac{1}{3}} \ \times \ x^{\frac{2}{3}} \\ &= \ 3 \ \times \ 4 \ \times \ x^{\frac{1}{3}+\frac{2}{3}} \\ &= \ 12 \ \times \ x^{\frac{3}{3}} \\ &= \ 12x^1 \\ &= \ 12x \end{aligned}$$

. . . . . .

Go online

Multiplication and division of indices practice

Q49: Simplify $x^6 \times x^4 \div x^5$

. . . . . .

Q50: Simplify $\frac{x^6 \times x^2}{x^3}, \ x \ \neq \ 0$

. . . . . .

Q51: Simplify $2x^{\frac{1}{4}} \ \times \ 5x^{\frac{1}{4}}$

. . . . . .

Go online

Multiplication and division of indices exercise

These questions are for practice only.

Q52: Simplify $\frac{x^4 \times x^3}{x^6}, \quad x \neq 0$

. . . . . .

Q53: Simplify $\frac{x^3 \times x^2}{x^4}, \quad x \neq 0$

. . . . . .

Q54: Simplify $\frac{x^6 \times 2x^5}{x^3}$, $x \neq 0$

.....

Q55: Simplify $2a^{\frac{1}{2}} \times 4a^{\frac{3}{2}}$

.....

1.5 Raising a power to a power

How to raise a power to a power

Go online

$(a^x)^y \quad if \quad x = 1 \quad and \quad y = 1$
$= (a^1)^1$
$= (a)^1$
$= a$

$(a^x)^y \quad if \quad x = 4 \quad and \quad y = 2$
$= (a^4)^2$
$= (a \times a \times a \times a)^2$
$= (a \times a \times a \times a)(a \times a \times a \times a)$
$= a^8$
$= a^{4\times 2}$

.....

Key point

We now have three laws of indices.

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{m\times n}$

Example

Problem:

Simplify $(a^3)^5$.

Solution:

$(a^3)^5 = a^{3\times 5} = a^{15}$

.....

Examples

1.

Problem:

Simplify $(2y^4)^2$.

Solution:

$(2y^4)^2 = 2^2 \times y^{4\times 2} = 4y^8$

.....

2.

Problem:

Simplify $(2g^{\frac{1}{2}})^6$.

Solution:

$(2g^{\frac{1}{2}})^6 = 2^6 \times g^{\frac{1}{2}\times 6} = 64g^{\frac{6}{2}} = 64g^3$

.....

Go online

Raising powers practice

Q56: Simplify $(a^2)^7$

.....

Q57: Simplify $(3y^4)^3$

.....

Q58: Simplify $(5m^{\frac{3}{2}})^2$

.....

Go online

Using three laws of indices exercise

Q59: Simplify $(a^6)^2$

.....

Q60: Simplify $(2b^3)^2$

.....

Q61: Simplify $(f^4)^2 \times f^3$

.....

Q62: Simplify $(4m^{\frac{1}{3}})^3$

.....

Q63: Simplify $(2n^{\frac{1}{2}})^2 \times (3n^5)^2$

.....

1.6 Negative and zero indices

Negative and zero indices


Go online

$If\ x = 0$

$a^0 = 1$

$If\quad x = -2$

$a^{-2} = \frac{1}{a \times a}$

$If\quad x = -5$

$a^{-5} = \frac{1}{a \times a \times a \times a \times a}$

. . . . . .

Key point

We now have five laws of indices.

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{m \times n}$
4. $a^0 = 1$
5. $a^{-m} = \frac{1}{a^m}$

Examples

1.

Problem:

Simplify, giving your answer with a positive index $a^{-5} \times a^4 \times a^{-3}$

Solution:

$a^{(-5)+4+(-3)} = a^{-4} = \frac{1}{a^4}$

. . . . . .

2.

Problem:

Simplify, giving your answer with a positive index $\frac{y^7}{y^{10}}$

Solution:

$\frac{y^7}{y^{10}} = y^{7-10} = y^{-3} = \frac{1}{y^3}$

. . . . . .

3.

Problem:

Simplify $(2g^{-2})^3$

Solution:

$(2g^{-2})^3 = 2^3 \times g^{-2\times 3} = 8g^{-6} = \frac{8}{g^6}$

.....

Negative and zero indices practice

Q64: Simplify, giving your answer with a positive index $\frac{a^{-3} \times a^5}{a^2}$

.....

Q65: Simplify, giving your answer with a positive index $(3m^{-4})^2$

.....

Q66: Simplify, giving your answer with a positive index $\frac{y^{-10} \times y^{\frac{3}{2}}}{y^{\frac{1}{2}}}$

.....

Go online

Using five laws of indices exercise

These exercises are for further practice of five laws of indices.

Q67: Simplify $a^{-2} \times a^2$

.....

Q68: Simplify $(t^{-5})^2$

.....

Q69: Simplify $(5b^{-7})^2$

.....

Q70: Simplify, giving your answer with a positive index $\frac{f^3 \times f^{-8}}{f^2}$

.....

Q71: Simplify, giving your answer with a positive index $\frac{2k^5 \times 3k^{-7}}{6k^4}$

.....

Q72: Simplify, giving your answer with a positive index $(2n^{\frac{1}{2}})^4 \times (10n^{-\frac{3}{2}})^2$

.....

1.7 Fractional indices

The purpose of a fractional index is to define a surd.

$a^{\frac{1}{2}} = \sqrt{a}$ $a^{\frac{1}{3}} = \sqrt[3]{a}$ $a^{\frac{1}{4}} = \sqrt[4]{a}$ $a^{\frac{3}{2}} = \sqrt{a^3}$ $a^{\frac{2}{3}} = \sqrt[3]{a^2}$

In essence the numerator of the index is the power and the denominator is the root.

Go online

Changing fractional indices

$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

If $x = 1$ and $y = 2$ then $a^{\frac{1}{2}} = \sqrt{a}$

If $x = 3$ and $y = 2$ then $a^{\frac{3}{2}} = \sqrt{a^3}$

If $x = 4$ and $y = 3$ then $a^{\frac{4}{3}} = \sqrt[3]{a^4}$

If $x = 4$ and $y = 4$ then $a^{\frac{4}{4}} = \sqrt[4]{a^4} = a$

. . . . . .

Key point

We now have six laws of indices.

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{m \times n}$
4. $a^0 = 1$
5. $a^{-m} = \frac{1}{a^m}$
6. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Examples

1.

Problem:

Evaluate $25^{\frac{1}{2}}$

Solution:

$25^{\frac{1}{2}} = \sqrt{25} = 5$

. . . . . .

2.

Problem:

Evaluate $25^{\frac{3}{2}}$

Solution:

$25^{\frac{3}{2}} = \sqrt[2]{25^3} = 5^3 = 125$

It does not matter whether you square root or cube the term first and since you probably don't know 25^3 it is easier to find $\sqrt{25}$ then cube the answer.

. . . . . .

3.

Problem:

Evaluate $8^{-\frac{1}{3}}$

Solution:

$8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

- Remember a negative index moves the term onto the denominator.

- The power a third means the cube root.
- The cube root of 8 is 2 because $2^3 = 8$

.....

Go online

Fractional indices practice

Q73: Evaluate $9^{\frac{1}{2}}$

.....

Q74: Evaluate $49^{\frac{1}{2}}$

.....

Q75: Evaluate $27^{\frac{1}{3}}$

.....

Q76: Evaluate $16^{\frac{1}{4}}$

.....

Q77: Evaluate $27^{\frac{2}{3}}$

.....

Q78: Evaluate $4^{\frac{3}{2}}$

.....

Go online

Using six laws of indices exercise

These exercises are for further practice of the six laws of indices.

Q79: Evaluate $81^{\frac{1}{2}}$

.....

Q80: Evaluate $64^{\frac{1}{3}}$

.....

Q81: Evaluate $32^{\frac{1}{5}}$

.....

Q82: Evaluate $100^{\frac{3}{2}}$

.....

Q83: Evaluate $1000^{\frac{2}{3}}$

.....

Q84: Evaluate $81^{\frac{3}{4}}$

.....

Q85: Evaluate $36^{-\frac{1}{2}}$

.....

Q86: Evaluate $16^{-\frac{3}{4}}$

.....

1.8 Learning points

To simplify surds.

- $\sqrt{63} = \sqrt{9} \times \sqrt{7} = 3\sqrt{7}$
- $\sqrt{\frac{32}{27}} = \frac{\sqrt{16}\times\sqrt{2}}{\sqrt{9}\times\sqrt{3}} = \frac{4\sqrt{2}}{3\sqrt{3}}$
- $6\sqrt{2} + 5\sqrt{2} - 3\sqrt{2} = 8\sqrt{2}$

To rationalise a denominator.

- $\frac{2}{\sqrt{14}} = \frac{2\times\sqrt{14}}{\sqrt{14}\times\sqrt{14}} = \frac{2\sqrt{14}}{14} = \frac{\sqrt{14}}{7}$

Six laws of indices.

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{m\times n}$
4. $a^0 = 1$
5. $a^{-m} = \frac{1}{a^m}$
6. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

1.9 End of topic test

Go online

End of topic 22 test

Q87:

i. Simplify $\sqrt{27}$
ii. Simplify $\sqrt{80}$
iii. Simplify $\sqrt{\frac{16}{4}}$
iv. Simplify $\sqrt{\frac{44}{100}}$
v. Simplify $\sqrt{\frac{28}{128}}$

.....

Q88:

i. Simplify $\sqrt{2} + 3\sqrt{2}$
ii. Simplify $6\sqrt{5} - 2\sqrt{5}$
iii. Simplify $3\sqrt{7} - 5\sqrt{7} + 4\sqrt{7}$

.....

Q89:

i. Rationalise the denominator $\frac{3}{\sqrt{7}}$
ii. Rationalise the denominator $\frac{2}{\sqrt{10}}$
iii. Rationalise the denominator $\frac{20}{\sqrt{20}}$
iv. Rationalise the denominator $\frac{1}{\sqrt{6}+1}$

.....

Q90:

i. Simplify $\frac{\left(x^5 \times x^6\right)}{x^2}, \quad x \neq 0.$
ii. Simplify $2a^{-\frac{1}{3}} \times 6a^{\frac{4}{3}}, \quad x \neq 0$

.....

Q91:

i. Simplify $(2g^4)^3$
ii. Simplify $\frac{(y^3)^5}{y^2}$

.....

Q92:

i. Simplify $\frac{a^7 \times (a^{-2})^2}{a^3}$
ii. Simplify, giving your answer as a positive index $(n^{\frac{1}{2}})^{-2} \times (n^{-\frac{3}{2}})^2$

. . . . . .

Q93:

i. Evaluate $121^{\frac{1}{2}}$
ii. Evaluate $16^{\frac{3}{2}}$
iii. Evaluate $8^{-\frac{1}{3}}$
iv. Evaluate $125^{-\frac{2}{3}}$

. . . . . .

Glossary

squares

Remember the squares or square numbers are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,....

Answers to questions and activities

22 Surds and indices

Simplifying surds: Rule for multiplication practice (page 4)

Q1: $\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

Q2: $\sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$

Q3: $\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

Q4: $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

Q5: $\sqrt{54} = \sqrt{9} \times \sqrt{6} = 3\sqrt{6}$

Q6: $\sqrt{80} = \sqrt{4} \times \sqrt{20} = \sqrt{4} \times \sqrt{4} \times \sqrt{5} = 2 \times 2 \times \sqrt{5} = 4\sqrt{5}$

Simplifying surds: Rule for division practice (page 6)

Q7: $\sqrt{\frac{80}{5}} = \sqrt{16} = 4$

Q8: $\frac{\sqrt{98}}{\sqrt{28}} = \frac{\sqrt{49}\times\sqrt{2}}{\sqrt{4}\times\sqrt{7}} = \frac{7\sqrt{2}}{2\sqrt{7}}$

Q9: $\frac{\sqrt{32}}{\sqrt{20}} = \frac{\sqrt{16}\times\sqrt{2}}{\sqrt{4}\times\sqrt{5}} = \frac{4\sqrt{2}}{2\sqrt{5}} = \frac{2\sqrt{2}}{\sqrt{5}}$

Q10: $\frac{\sqrt{200}}{\sqrt{50}} = \frac{\sqrt{100}\times\sqrt{2}}{\sqrt{25}\times\sqrt{2}} = \frac{10\sqrt{2}}{5\sqrt{2}} = \frac{10}{5} = 2$

Q11: $\sqrt{\frac{45}{12}} = \frac{\sqrt{9}\times\sqrt{5}}{\sqrt{4}\times\sqrt{3}} = \frac{3\sqrt{5}}{2\sqrt{3}}$

Q12: $\sqrt{\frac{54}{30}} = \frac{\sqrt{9}\times\sqrt{6}}{\sqrt{5}\times\sqrt{6}} = \frac{\sqrt{9}}{\sqrt{5}} = \frac{3}{\sqrt{5}}$

Simplifying surds exercise (page 6)

Q13: $3\sqrt{3}$

Q14: $4\sqrt{5}$

Q15: $10\sqrt{3}$

Q16: $2\sqrt{11}$

Q17: $7\sqrt{2}$

Q18: $5\sqrt{5}$

Q19: $\frac{4}{3}$

Q20: $\frac{2\sqrt{6}}{7}$

Q21: $\frac{9}{5\sqrt{5}}$

Q22: $\frac{4}{3}$

Q23: $\frac{5\sqrt{3}}{6\sqrt{2}}$

Collecting like terms practice: Addition and subtraction (page 7)

Q24: $5\sqrt{3}$

Q25: $11\sqrt{2}$

Q26: $4\sqrt{3}$

Q27: $\sqrt{5}$

Q28: $4\sqrt{7}$

Combining simplifying surds and collecting like terms practice (page 7)

Q29: $\sqrt{9} \times \sqrt{2} + \sqrt{2} = 3\sqrt{2} + \sqrt{2} = 4\sqrt{2}$

Q30: $7\sqrt{3} - \sqrt{9} \times \sqrt{3} = 7\sqrt{3} - 3\sqrt{3} = 4\sqrt{3}$

Q31: $\sqrt{16} \times \sqrt{2} - 3\sqrt{2} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

Q32: $2\sqrt{5} + \sqrt{9} \times \sqrt{5} - \sqrt{5} = 2\sqrt{5} + 3\sqrt{5} - \sqrt{5} = 4\sqrt{5}$

Collecting like terms exercise (page 8)

Q33: $3\sqrt{3}$

Q34: $3\sqrt{6}$

Q35: $6\sqrt{5}$

Q36: $3\sqrt{7}$

Q37: $8\sqrt{2}$

Rationalise the denominator practice (page 9)

Q38: $\frac{1}{\sqrt{6}} = \frac{1\times\sqrt{6}}{\sqrt{6}\times\sqrt{6}} = \frac{\sqrt{6}}{\sqrt{36}} = \frac{\sqrt{6}}{6}$

Q39: $\frac{3}{2\sqrt{3}} = \frac{3\times\sqrt{3}}{2\sqrt{3}\times\sqrt{3}} = \frac{3\sqrt{3}}{2\sqrt{9}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

Q40: $\frac{\sqrt{2}+1}{\sqrt{10}} = \frac{(\sqrt{2}+1)\times\sqrt{10}}{\sqrt{10}\times\sqrt{10}} = \frac{\sqrt{20}+\sqrt{10}}{\sqrt{100}} = \frac{\sqrt{4}\times\sqrt{5}+\sqrt{10}}{10} = \frac{2\sqrt{5}+\sqrt{10}}{10}$

Q41: $\frac{1}{\sqrt{5}-1} = \frac{1\times(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} = \frac{\sqrt{5}+1}{\sqrt{5}^2-1^2} = \frac{\sqrt{5}+1}{5-1} = \frac{\sqrt{5}+1}{4}$

Rationalising denominators exercise (page 10)

Q42: $\frac{\sqrt{7}}{7}$

Q43: $\frac{\sqrt{13}}{13}$

Q44: $2\sqrt{2}$

Q45: $\frac{\sqrt{14}}{7}$

Q46: $\frac{\sqrt{10}}{4}$

Q47: $\frac{\sqrt{5}+5}{5}$

Q48: $\frac{\sqrt{3}-1}{2}$

Multiplication and division of indices practice (page 12)

Q49: $x^6 \times x^4 \div x^5 = x^{6+4-5} = x^5$

Q50: $\frac{x^6 \times x^2}{x^3} = x^{6+2-3} = x^5$

Q51:

$$\begin{aligned} 2x^{\frac{1}{4}} \times 5x^{\frac{1}{4}} &= 2 \times 5 \times x^{\frac{1}{4}} \times x^{\frac{1}{4}} \\ &= 2 \times 5 \times x^{\frac{1}{4}+\frac{1}{4}} \\ &= 10 \times x^{\frac{2}{4}} \\ &= 10x^{\frac{1}{2}} \end{aligned}$$

Multiplication and division of indices exercise (page 12)

Q52: $x^1 = x$

Q53: $x^1 = x$

Q54: $2x^8$

Q55: $8a^2$

Raising powers practice (page 14)

Q56: $(a^2)^7 = a^{2\times 7} = a^{14}$

Q57: $(3y^4)^3 = 3^3 \times y^{4\times 3} = 27y^{12}$

Q58: $(5m^{\frac{3}{2}})^2 = 5^2 \times m^{\frac{3}{2}\times 2} = 25m^{\frac{6}{2}} = 25m^3$

Using three laws of indices exercise (page 14)

Q59: a^{12}

Q60: $4b^6$

Q61: f^{11}

Q62: $64m$

Q63: $36n^{11}$

Negative and zero indices practice (page 16)

Q64: $a^{(-3)\,+\,5\,-\,2} \;=\; a^0 \;=\; 1$

Q65: $(3m^{-4})^2 = 3^2 \times m^{(-4)\times 2} = 9m^{-8} = \frac{9}{m^8}$

Q66: $y^{(-10)+\frac{3}{2}-\frac{1}{2}} = y^{(-10)+1} = y^{-9} = \frac{1}{y^9}$

Using five laws of indices exercise (page 16)

Q67: 1

Q68: t^{-10}

Q69: $25b^{-14}$

Q70: $\frac{1}{f^7}$

Q71: $\frac{1}{k^6}$

Q72: $16n^2 \times 100n^{-3} = \frac{1600}{n}$

Fractional indices practice (page 18)

Q73: $9^{\frac{1}{2}} = \sqrt{9} = 3$

Q74: $49^{\frac{1}{2}} = \sqrt{49} = 7$

Q75: $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ because $3^3 = 27$

Q76: $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$ because $2^4 = 16$

Q77: $27^{\frac{2}{3}} = \sqrt[3]{27^2} = 9$ because $\sqrt[3]{27} = 3$ and $3^2 = 9$

Q78: $4^{\frac{3}{2}} = \sqrt{4^3} = 8$ because $\sqrt{4} = 2$ and $2^3 = 8$

Using six laws of indices exercise (page 18)

Q79: 9

Q80: 4

Q81: 2

Q82: 1000

Q83: 100

Q84: 27

Q85: $\frac{1}{6}$

Q86: $\frac{1}{8}$

End of topic 22 test (page 20)

Q87:

i. $3\sqrt{3}$
ii. $4\sqrt{5}$
iii. 2
iv. $\frac{\sqrt{11}}{5}$
v. $\frac{\sqrt{7}}{4\sqrt{2}}$

Q88:

i. $4\sqrt{2}$
ii. $4\sqrt{5}$
iii. $2\sqrt{7}$

Q89:

i. $\frac{3\sqrt{7}}{7}$
ii. $\frac{\sqrt{10}}{5}$
iii. $2\sqrt{5}$
iv. $\frac{\sqrt{6}-1}{5}$

Q90:

i. x^9
ii. $12a$

Q91:

i. $8g^{12}$
ii. y^{13}

Q92:

i. 1

ii. $\frac{1}{n^4}$

Q93:

i. 11

ii. 64

iii. $\frac{1}{2}$

iv. $\frac{1}{25}$