

Nat 5

USAP 4(a)



This booklet contains :

- Questions on Topics covered in RHS USAP 4(a)
- Exam Type Questions
- Answers

Sourced from PEGASYS

USAP 4(a)

EF 1.1 Working with surds

- **Simplification**
- **Rationalising denominators**

EF 1.2 Simplifying expressions

- **Multiplication and division using positive and negative indices including fractions**
- **Calculations using Scientific Notation**

REL 4.1 Working with trigonometric functions

- **Basic curves**
- **Scaling amplitude**
- **Vertical translation**
- **Multiple and phase angles**

EF 1.1 WORKING with SURDS

1. Express each of the following in its simplest form:

- (a) $\sqrt{8}$ (b) $\sqrt{12}$ (c) $\sqrt{50}$ (d) $\sqrt{20}$ (e) $\sqrt{24}$ (f) $\sqrt{108}$
(g) $\sqrt{60}$ (h) $\sqrt{72}$ (i) $\sqrt{300}$ (j) $\sqrt{27}$ (k) $\sqrt{96}$ (l) $\sqrt{48}$
(m) $\sqrt{45}$ (n) $\sqrt{98}$ (o) $\sqrt{90}$ (p) $\sqrt{18}$ (q) $\sqrt{28}$ (r) $\sqrt{80}$
(s) $\sqrt{32}$ (t) $\sqrt{160}$ (u) $\sqrt{150}$ (v) $\sqrt{44}$ (w) $\sqrt{63}$ (x) $\sqrt{175}$

2. Simplify:

- (a) $5\sqrt{8}$ (b) $3\sqrt{32}$ (c) $5\sqrt{40}$ (d) $2\sqrt{12}$ (e) $4\sqrt{18}$ (f) $3\sqrt{24}$
(g) $3\sqrt{27}$ (h) $10\sqrt{48}$ (i) $2\sqrt{108}$ (j) $3\sqrt{45}$ (k) $2\sqrt{63}$ (l) $4\sqrt{20}$

3. Express each of the following in its simplest form:

- (a) $5\sqrt{2} + 3\sqrt{2}$ (b) $3\sqrt{7} - \sqrt{7}$ (c) $4\sqrt{3} + 2\sqrt{3} - 3\sqrt{3}$
(d) $5\sqrt{6} - 2\sqrt{6} + \sqrt{6}$ (e) $4\sqrt{3} + 5\sqrt{3}$ (f) $8\sqrt{6} - 2\sqrt{6}$
(g) $\sqrt{2} + 2\sqrt{2}$ (h) $3\sqrt{7} - 9\sqrt{7}$ (i) $5\sqrt{10} - 5\sqrt{10}$
(j) $\sqrt{5} + 5\sqrt{5} - 3\sqrt{5}$ (k) $2\sqrt{3} + \sqrt{3} - 5\sqrt{3}$ (l) $5\sqrt{11} + 7\sqrt{11} - \sqrt{11}$

4. Express each of the following in its simplest form:

- (a) $\sqrt{12} + \sqrt{27}$ (b) $\sqrt{32} - \sqrt{8}$ (c) $\sqrt{72} - \sqrt{50}$
(d) $\sqrt{2} + \sqrt{98}$ (e) $\sqrt{80} + \sqrt{20}$ (f) $\sqrt{24} + \sqrt{54}$
(g) $\sqrt{180} - \sqrt{45}$ (h) $\sqrt{1000} - \sqrt{90}$ (i) $\sqrt{50} - \sqrt{8}$
(j) $\sqrt{3} - \sqrt{12}$ (k) $\sqrt{75} + \sqrt{108} - \sqrt{3}$ (l) $\sqrt{5} + \sqrt{20} + \sqrt{80}$
(m) $\sqrt{108} + \sqrt{12}$ (n) $\sqrt{32} - \sqrt{8}$ (o) $\sqrt{72} - \sqrt{50}$
(p) $\sqrt{2} + \sqrt{98}$ (q) $\sqrt{80} + \sqrt{20}$ (r) $\sqrt{24} + \sqrt{54}$
(s) $\sqrt{8} + 5\sqrt{2}$ (t) $3\sqrt{12} + \sqrt{27}$ (u) $3\sqrt{2} + 2\sqrt{8} - \sqrt{18}$

5. Simplify:

(a) $\sqrt{5} \times \sqrt{5}$

(b) $\sqrt{2} \times \sqrt{2}$

(c) $\sqrt{11} \times \sqrt{11}$

(d) $\sqrt{a} \times \sqrt{a}$

(e) $\sqrt{6} \times \sqrt{6}$

(f) $\sqrt{c} \times \sqrt{c}$

(g) $\sqrt{k} \times \sqrt{k}$

(h) $\sqrt{3} \times \sqrt{6}$

(i) $\sqrt{8} \times \sqrt{2}$

(j) $\sqrt{6} \times \sqrt{2}$

(k) $\sqrt{3} \times \sqrt{5}$

(l) $\sqrt{x} \times \sqrt{y}$

(m) $\sqrt{2} \times \sqrt{8}$

(n) $\sqrt{12} \times \sqrt{3}$

(o) $\sqrt{5} \times \sqrt{20}$

(p) $\sqrt{2} \times \sqrt{32}$

(q) $\sqrt{a} \times \sqrt{b}$

(r) $\sqrt{10} \times \sqrt{x}$

(s) $\sqrt{p} \times \sqrt{q}$

(t) $\sqrt{k} \times \sqrt{6}$

(u) $\sqrt{2} \times \sqrt{10}$

(v) $\sqrt{24} \times \sqrt{3}$

(w) $\sqrt{5} \times \sqrt{10}$

(x) $\sqrt{6} \times \sqrt{12}$

(y) $\sqrt{20} \times \sqrt{3}$

(z) $\sqrt{4} \times \sqrt{8}$

6. (a) $3\sqrt{2} \times \sqrt{2}$ (b) $2\sqrt{5} \times 3\sqrt{5}$ (c) $3\sqrt{2} \times 2\sqrt{7}$ (d) $4\sqrt{3} \times 2\sqrt{3}$

(e) $\sqrt{5} \times 3\sqrt{2}$ (f) $2\sqrt{6} \times 3\sqrt{3}$ (g) $8\sqrt{2} \times \sqrt{12}$ (h) $5\sqrt{3} \times 3\sqrt{5}$

7. Simplify:

(a) $\frac{\sqrt{8}}{\sqrt{2}}$

(b) $\frac{\sqrt{27}}{\sqrt{12}}$

(c) $\frac{\sqrt{2}}{\sqrt{32}}$

(d) $\frac{\sqrt{3}}{\sqrt{27}}$

(e) $\frac{\sqrt{20}}{\sqrt{5}}$

(f) $\frac{\sqrt{12}}{\sqrt{48}}$

(g) $\frac{\sqrt{54}}{\sqrt{24}}$

(h) $\frac{\sqrt{175}}{\sqrt{63}}$

(i) $\frac{\sqrt{18}}{\sqrt{72}}$

(j) $\frac{\sqrt{6}}{\sqrt{54}}$

(k) $\frac{\sqrt{288}}{\sqrt{8}}$

(l) $\frac{\sqrt{1000}}{\sqrt{90}}$

(m) $\frac{\sqrt{48}}{\sqrt{6}}$

(n) $\frac{\sqrt{3}}{\sqrt{24}}$

(o) $\frac{\sqrt{98}}{\sqrt{7}}$

(p) $\frac{\sqrt{50}}{\sqrt{250}}$

8. Expand and simplify:

- | | | |
|--------------------------------------|---------------------------------------|--|
| (a) $\sqrt{2}(1 - \sqrt{2})$ | (b) $\sqrt{3}(\sqrt{3} + 1)$ | (c) $\sqrt{5}(\sqrt{5} - 1)$ |
| (d) $\sqrt{2}(5 + \sqrt{2})$ | (e) $\sqrt{2}(3 + \sqrt{6})$ | (f) $2\sqrt{3}(\sqrt{8} + 1)$ |
| (g) $\sqrt{3}(\sqrt{6} - 2\sqrt{8})$ | (h) $\sqrt{5}(\sqrt{5} + 2)$ | (i) $4\sqrt{6}(2\sqrt{6} - \sqrt{8})$ |
| (j) $\sqrt{8}(\sqrt{2} + 4)$ | (k) $2\sqrt{12}(\sqrt{3} + \sqrt{6})$ | (l) $\sqrt{5}(\sqrt{200} + \sqrt{50})$ |
| (m) $\sqrt{3}(\sqrt{2} + 1)$ | (n) $\sqrt{2}(\sqrt{8} + \sqrt{2})$ | (o) $\sqrt{3}(\sqrt{2} + \sqrt{6})$ |
| (p) $\sqrt{5}(3 - \sqrt{5})$ | | |

9. Expand and simplify where possible:

- | | | |
|-------------------------------------|-------------------------------------|--|
| (a) $(\sqrt{2} + 3)(\sqrt{2} - 1)$ | (b) $(\sqrt{5} + 1)(2\sqrt{5} - 4)$ | (c) $(2\sqrt{2} + 3)(\sqrt{2} + 4)$ |
| (d) $(\sqrt{3} + 1)(\sqrt{3} - 1)$ | (e) $(2 + \sqrt{5})(2 - \sqrt{5})$ | (f) $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$ |
| (g) $(\sqrt{2} - 4)(3\sqrt{2} - 1)$ | (h) $(\sqrt{8} + 2)(\sqrt{8} + 1)$ | (i) $(2\sqrt{3} + \sqrt{2})(\sqrt{3} + 3\sqrt{2})$ |
| (j) $(\sqrt{2} + 3)^2$ | (k) $(\sqrt{2} + \sqrt{3})^2$ | (l) $(2\sqrt{3} - 1)^2$ |
| (m) $(2\sqrt{7} - \sqrt{2})^2$ | (n) $(5 - 2\sqrt{3})^2$ | (o) $(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})$ |
| (p) $(\sqrt{7} + 1)^2$ | (q) $(\sqrt{6} + \sqrt{2})^2$ | (r) $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$ |

10. Express each of the following with a *rational denominator* and simplify where possible:

- | | | | |
|---------------------------|---------------------------|--------------------------|---------------------------|
| (a) $\frac{1}{\sqrt{2}}$ | (b) $\frac{1}{\sqrt{3}}$ | (c) $\frac{1}{\sqrt{5}}$ | (d) $\frac{6}{\sqrt{3}}$ |
| (e) $\frac{10}{\sqrt{5}}$ | (f) $\frac{2}{\sqrt{3}}$ | (g) $\frac{3}{\sqrt{5}}$ | (h) $\frac{20}{\sqrt{2}}$ |
| (i) $\frac{2}{\sqrt{2}}$ | (j) $\frac{12}{\sqrt{3}}$ | (k) $\frac{3}{\sqrt{6}}$ | (l) $\frac{4}{\sqrt{5}}$ |
| (m) $\frac{10}{\sqrt{2}}$ | (n) $\frac{35}{\sqrt{7}}$ | | |

11. Express each of the following with a *rational denominator* and simplify where possible:

(a) $\frac{1}{2\sqrt{5}}$ (b) $\frac{4}{5\sqrt{2}}$ (c) $\frac{3}{3\sqrt{2}}$ (d) $\frac{12}{5\sqrt{6}}$

(e) $\frac{8}{3\sqrt{2}}$ (f) $\frac{20}{7\sqrt{5}}$ (g) $\frac{50}{3\sqrt{10}}$ (h) $\frac{10}{3\sqrt{2}}$

12. Express each of the following in its simplest form with a rational denominator.

(a) $\frac{\sqrt{3}}{\sqrt{2}}$ (b) $\frac{\sqrt{2}}{\sqrt{5}}$ (c) $\frac{\sqrt{8}}{\sqrt{2}}$ (d) $\frac{\sqrt{18}}{\sqrt{3}}$

(e) $\frac{\sqrt{5}}{\sqrt{20}}$ (f) $\frac{\sqrt{2}}{\sqrt{12}}$ (g) $\frac{\sqrt{15}}{\sqrt{5}}$ (h) $\frac{\sqrt{8}}{\sqrt{6}}$

(i) $\frac{\sqrt{5}}{\sqrt{2}}$ (j) $\frac{\sqrt{11}}{\sqrt{2}}$ (k) $\frac{\sqrt{7}}{\sqrt{3}}$ (l) $\frac{\sqrt{13}}{\sqrt{5}}$

(m) $\frac{\sqrt{8}}{3\sqrt{2}}$ (n) $\frac{2\sqrt{3}}{3\sqrt{2}}$ (o) $\frac{5\sqrt{3}}{3\sqrt{5}}$ (p) $\frac{4\sqrt{5}}{5\sqrt{3}}$

(q) $\frac{\sqrt{6}}{\sqrt{18}}$ (r) $\frac{\sqrt{50}}{\sqrt{10}}$ (s) $\frac{\sqrt{3}}{\sqrt{12}}$ (l) $\frac{\sqrt{5}}{\sqrt{2}}$

13. Express each of the following with a *rational denominator* and simplify where possible:

(a) $\frac{1}{\sqrt{50}}$ (b) $\frac{18}{\sqrt{27}}$ (c) $\frac{5}{\sqrt{50}}$ (d) $\frac{3}{\sqrt{20}}$

(e) $\frac{6}{\sqrt{18}}$ (f) $\frac{2}{\sqrt{8}}$ (g) $\frac{10}{\sqrt{12}}$ (h) $\frac{3}{\sqrt{50}}$

(i) $\frac{4}{\sqrt{32}}$ (j) $\frac{2\sqrt{3}}{\sqrt{54}}$ (k) $\frac{3\sqrt{2}}{\sqrt{24}}$ (l) $\frac{2\sqrt{5}}{\sqrt{45}}$

14. Rationalise the denominator, in each fraction, using the appropriate conjugate surd.

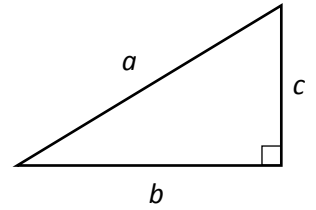
(a)	$\frac{1}{\sqrt{2}-1}$	(b)	$\frac{1}{\sqrt{5}+1}$	(c)	$\frac{12}{2-\sqrt{3}}$	(d)	$\frac{1}{1-\sqrt{2}}$
(e)	$\frac{1}{1+\sqrt{3}}$	(f)	$\frac{3}{\sqrt{5}-1}$	(g)	$\frac{2}{\sqrt{2}+2}$	(h)	$\frac{3}{2-\sqrt{6}}$
(i)	$\frac{5}{3+\sqrt{2}}$	(j)	$\frac{4}{1+\sqrt{3}}$	(k)	$\frac{1}{\sqrt{7}-2}$	(l)	$\frac{1}{\sqrt{3}-\sqrt{2}}$
(m)	$\frac{6}{\sqrt{3}+\sqrt{2}}$	(n)	$\frac{12}{\sqrt{10}-\sqrt{2}}$	(o)	$\frac{3}{\sqrt{5}+\sqrt{6}}$	(p)	$\frac{14}{9-\sqrt{2}}$

SURDS

PROBLEMS

1. A right angled triangle has sides a , b and c as shown.

For each case below calculate the length of the third side, expressing your answer as a surd in its simplest form.



- (a) Find a if $b = 6$ and $c = 3$. (b) Find c if $a = 2$ and $b = 1$.
(c) Find c if $a = 18$ and $b = 12$ (d) Find b if $a = 2\sqrt{8}$ and $c = 2\sqrt{6}$.

2. Given that $x = 1 + \sqrt{2}$ and $y = 1 - \sqrt{2}$, simplify:

- (a) $5x + 5y$ (b) $2xy$ (c) $x^2 + y^2$ (d) $(x + y)(x - y)$

3. Given that $p = \sqrt{5} + \sqrt{3}$ and $q = \sqrt{5} - \sqrt{3}$, simplify:

- (a) $2p - 2q$ (b) $4pq$ (c) $p^2 - q^2$

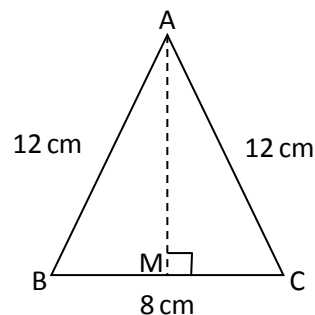
4. A rectangle has sides measuring $(2 + \sqrt{2})$ cm and $(2 - \sqrt{2})$ cm.

Calculate the exact value of (a) its area (b) the length of a diagonal.

5. A curve has as its equation $y = 2 + \frac{1}{2}x^2$.

- (a) If the point $P(\sqrt{2}, k)$ lies on this curve find the exact value of k .
(b) Find the exact length of OP where O is the origin.

6. In $\triangle ABC$, $AB = AC = 12\text{cm}$ and $BC = 8\text{cm}$. Express the length of the altitude from A to BC as a surd in its simplest form. [The line AM in the diagram]



7. An equilateral triangle has each of its sides measuring $2a$ metres.
- (a) Find the exact length of an altitude of the triangle in terms of a .
- (b) Hence find the exact area of the triangle in terms of a .

[Draw a diagram to help you with this question]

8. The exact **area** of a rectangle is $2(\sqrt{6} + \sqrt{3})$ square centimetres. Given that the breadth of the rectangle is $\sqrt{6}$ cm, show that the length is equal to $(2 + \sqrt{2})$ cm.

9. (a challenge) Given that $\tan 75^\circ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$, show that $\tan 75^\circ = 2 + \sqrt{3}$.

EF 1.2 INDICES

1. Write each of the following in its simplest index form.

(a) $3^4 \times 3^2$	(b) 2×2^3	(c) $10^5 \times 10^2$	(d) $8^3 \times 8^5$
(e) $7^6 \times 7$	(f) $5^4 \times 5^4$	(g) $9^6 \times 9^2$	(h) $6^8 \times 6^5$
(i) $x^3 \times x^5$	(j) $c^2 \times c^9$	(k) $a^2 \times a^{12}$	(l) $y^5 \times y^5$
(m) $b^{10} \times b^{30}$	(n) $p \times p^9$	(o) $d^2 \times d^4$	(p) $q^{11} \times q^9$
(q) $t^3 \times t^7$	(r) $f^4 \times f^3$	(s) $k \times k^{12}$	(t) $z^{50} \times z^{50}$
(u) $x^{30} \times x^{50}$	(v) $y^{19} \times y$	(w) $a^{25} \times a^{65}$	(x) $b^1 \times b^0$

2. Write each of the following in its simplest index form.

(a) $2^8 \div 2^3$	(b) $5^4 \div 5^2$	(c) $12^9 \div 12^6$	(d) $7^{11} \div 7^4$
(e) $20^5 \div 20$	(f) $8^8 \div 8^4$	(g) $3^{18} \div 3^3$	(h) $4^{15} \div 4^{13}$
(i) $x^7 \div x^2$	(j) $a^9 \div a^5$	(k) $y^{20} \div y^{10}$	(l) $b^4 \div b^1$
(m) $p^{12} \div p^{11}$	(n) $c^7 \div c^7$	(o) $q^8 \div q^2$	(p) $d^4 \div d$
(q) $\frac{x^9}{x^3}$	(r) $\frac{a^8}{a^2}$	(s) $\frac{m^{14}}{m}$	(t) $\frac{s^7}{s^7}$
(u) $\frac{d^{20}}{d^{12}}$	(v) $\frac{y^{100}}{y^{10}}$	(w) $\frac{t^{100}}{t}$	(x) $\frac{w^{10}}{w^0}$

3. Write each of the following in its simplest index form.

(a) $(3^2)^4$	(b) $(8^2)^2$	(c) $(10^3)^2$	(d) $(2^2)^5$
(e) $(4^5)^3$	(f) $(1^7)^2$	(g) $(12^3)^3$	(h) $(5^5)^5$
(i) $(x^4)^2$	(j) $(y^8)^5$	(k) $(a^3)^7$	(l) $(m^4)^4$
(m) $(b^3)^6$	(n) $(p^5)^3$	(o) $(k^5)^{20}$	(p) $(z^6)^0$

4. Write the following without brackets.

- (a) $(2b)^2$ (b) $(7a)^3$ (c) $(3x)^4$ (d) $(2y)^5$
(e) $(ab)^4$ (f) $(xy)^7$ (g) $(wz)^5$ (h) $(st)^3$
(i) $(pq^2)^3$ (j) $(x^4y)^2$ (k) $(a^2b^3)^5$ (l) $(6a^5)^2$
(m) $(10x^2)^3$ (n) $(2c^4)^5$ (o) $(3ab^2)^3$ (p) $(4m^2k)^2$

5. Simplify these expressions.

- (a) $2a^3 \times 5a^5$ (b) $7x \times 9x^8$ (c) $12p^7 \div 4p^4$ (d) $50b^{12} \div 10b^6$
(e) $3y \times (2y^2)^3$ (f) $(4q^3)^2 \times 5q^4$ (g) $(4c^3)^3 \div 8c^2$ (h) $72z^{12} \div (3z^4)^2$
(i) $k^2(k^3 + k^5)$ (j) $m^5(m^2 - m^3)$ (k) $2x^4(x^3 + 3x^2)$ (l) $5a^5(2a^2 - 3a^3)$
(m) $\frac{x^5 \times x^4}{x^6}$ (n) $\frac{(m^5)^4}{m^6}$ (o) $\frac{5c^3 \times 4c^7}{2c^6}$ (p) $\frac{(3q^3)^2 \times 4q^4}{6q^7}$
(q) $\frac{(3xy^5)^3}{9x^2y}$ (r) $\frac{(2a^2b^5)^6}{(4ab)^2}$ (s) $\frac{(4p^4)^3}{2p^3 \times 8p^6}$ (t) $\frac{(2ab^3)^5}{3a^2b \times 4ab^2}$

6. Write down the value of

- (a) 5^0 (b) 2^0 (c) 100^0 (d) $(-3)^0$ (e) 25^0
(f) $\frac{1}{2}^0$ (g) a^0 (h) k^0 (i) $(mn)^0$ (j) $(ab^2)^0$
(k) $(10x^3)^0$ (l) $(16y^2z^3)^0$

7. Rewrite the following with positive indices.

- (a) 3^{-2} (b) 5^{-4} (c) 2^{-6} (d) 10^{-3} (e) 4^{-5} (f) 200^{-7}
(g) a^{-5} (h) x^{-2} (i) p^{-7} (j) y^{-10} (k) $2b^{-3}$ (l) $10q^{-x}$
(m) $\frac{1}{x^{-3}}$ (n) $\frac{1}{w^{-5}}$ (o) $\frac{3}{a^{-2}}$ (p) $\frac{10}{c^{-8}}$ (q) $\frac{2}{3t^{-1}}$ (r) $\frac{5}{4y^{-3}}$

8. Rewrite the following with negative indices.

(a) $\frac{1}{3^2}$ (b) $\frac{1}{6^9}$ (c) $\frac{1}{5^4}$ (d) $\frac{1}{2^7}$ (e) $\frac{1}{10^3}$ (f) $\frac{1}{4^4}$

(g) $\frac{1}{x^3}$ (h) $\frac{1}{a^5}$ (i) $\frac{1}{p^4}$ (j) $\frac{1}{y^{10}}$ (k) $\frac{1}{q^6}$ (l) $\frac{1}{c^8}$

9. Simplify the following expressions.

(a) $m^3 \times m^{-5}$ (b) $x^7 \times x^{-2}$ (c) $p^{-8} \times p^5$ (d) $a^{-3} \times a^{-5}$

(e) $(y^3)^{-4}$ (f) $(c^{-5})^3$ (g) $(q^3)^{-5}$ (h) $(w^{-2})^{-4}$

(i) $4b^{-4} \times 5b^5$ (j) $3x^6 \times 9x^{-6}$ (k) $4k^3 \div 2k^{-2}$ (l) $18d \div 12d^4$

(m) $x^2(x^3 + x^{-1})$ (n) $p^{-3}(p^4 - p^{-8})$ (o) $3a^5(2a + 3a^{-2})$

(p) $\frac{1}{2}m^{-2}(4m^{-3} - 10m^6)$ (q) $\frac{v^3 \times v^5}{v^{-2}}$ (r) $\frac{4h^7 \times 3h^{-4}}{2h^4}$

(s) $\frac{4c^{-5} \times 9c^6}{6c^{-4}}$ (t) $\frac{5x^4 \times 6x^{-8}}{3x^{-4}}$

10. Find the value of

(a) $16^{\frac{1}{4}}$ (b) $8^{\frac{1}{3}}$ (c) $36^{\frac{1}{2}}$ (d) $27^{\frac{2}{3}}$ (e) $64^{\frac{1}{3}}$ (f) $1000^{\frac{1}{3}}$

(g) $25^{\frac{1}{2}}$ (h) $81^{\frac{3}{4}}$ (i) $125^{\frac{2}{3}}$ (j) $64^{\frac{1}{2}}$ (k) $216^{\frac{1}{3}}$ (l) $16^{-\frac{1}{4}}$

(m) $4^{\frac{1}{2}}$ (n) $16^{-\frac{1}{2}}$ (o) $9^{\frac{1}{2}}$ (p) $27^{-\frac{2}{3}}$ (q) $256^{-\frac{3}{4}}$ (r) $1000^{-\frac{2}{3}}$

(s) $16^{-\frac{3}{2}}$ (t) $8^{\frac{4}{3}}$ (u) $8^{\frac{4}{3}}$ (v) $(-8)^{\frac{1}{3}}$ (w) $64^{\frac{2}{3}}$ (x) $100^{-\frac{3}{2}}$

(y) $(\frac{1}{2})^{-1}$ (z) $(\frac{1}{8})^{\frac{4}{3}}$

11. Simplify the following expressions, giving your answers with positive indices.

(a)	$(x^2)^6$	(b)	$(p^3)^6$	(c)	$(a^{\frac{3}{4}})^8$	(d)	$(y^{-\frac{2}{3}})^9$
(e)	$(q^{\frac{1}{5}})^{10}$	(f)	$(k^{\frac{2}{5}})^1$	(g)	$(g^4)^{\frac{1}{2}}$	(h)	$(m^{12})^{\frac{2}{3}}$
(i)	$(c^9)^{\frac{2}{3}}$	(j)	$(h^5)^{\frac{1}{2}}$	(k)	$(z^4)^{\frac{3}{4}}$	(l)	$(b^{16})^{\frac{3}{4}}$
(m)	$x^{\frac{1}{2}} \times x^{-\frac{1}{2}}$	(n)	$y^{\frac{1}{3}} \times y^{\frac{2}{3}}$	(o)	$d^{-\frac{1}{4}} \times d^{\frac{9}{4}}$	(p)	$s^{\frac{7}{2}} \times s^{-\frac{1}{2}}$
(q)	$3x^{\frac{1}{2}} \times 4x^{\frac{1}{2}}$	(r)	$6x^{\frac{1}{2}} \times 2x^{-\frac{1}{2}}$	(s)	$2x^{\frac{1}{2}} \times 5x^{\frac{1}{2}}$	(t)	$3x^{\frac{2}{3}} \times 2x^{-\frac{1}{3}}$
(u)	$x^{\frac{1}{2}} \div x^{\frac{1}{2}}$	(v)	$2x^{\frac{1}{2}} \div x^{-\frac{1}{2}}$	(w)	$8x^{\frac{2}{3}} \div 2x^{\frac{1}{3}}$	(x)	$6x^{\frac{1}{3}} \div 4x^{\frac{2}{3}}$

12. Write the following in surd form.

(a)	$x^{\frac{1}{2}}$	(b)	$y^{\frac{1}{3}}$	(c)	$a^{\frac{1}{4}}$	(d)	$y^{\frac{2}{3}}$
(e)	$b^{\frac{3}{4}}$	(f)	$x^{\frac{5}{3}}$	(g)	$c^{\frac{3}{5}}$	(h)	$a^{\frac{4}{5}}$
(i)	$c^{\frac{1}{3}}$	(j)	$z^{-\frac{1}{2}}$	(k)	$m^{-\frac{2}{3}}$	(l)	$k^{-\frac{3}{5}}$
(m)	$p^{-\frac{4}{3}}$	(n)	$x^{-\frac{5}{3}}$	(o)	$w^{-\frac{4}{5}}$	(p)	$d^{-\frac{2}{7}}$

13. Write the following in index form.

(a)	\sqrt{x}	(b)	$\sqrt[3]{a}$	(c)	$\sqrt{y^3}$	(d)	$\sqrt[3]{z^2}$
(e)	$\sqrt[3]{c^2}$	(f)	$\sqrt[4]{x^3}$	(g)	$\sqrt[3]{p^5}$	(h)	$\sqrt[5]{m^2}$
(i)	$\frac{1}{\sqrt{a}}$	(j)	$\frac{1}{\sqrt[3]{z}}$	(k)	$\frac{1}{\sqrt[3]{x^4}}$	(l)	$\frac{1}{\sqrt{a^5}}$
(m)	$\frac{1}{\sqrt[3]{b^2}}$	(n)	$\frac{1}{\sqrt[5]{m^3}}$	(o)	$\frac{1}{\sqrt[4]{y}}$	(p)	$\frac{1}{\sqrt[3]{c^5}}$

14. Simplify each of the following by (i) changing root signs to fractional powers;
(ii) moving x 's onto the numerators;
(iii) expanding brackets where necessary.

(a) $x^{\frac{1}{2}}(x^4 + 1)$ (b) $x^{-\frac{1}{2}}(x^{\frac{3}{2}} - x^2)$ (c) $\frac{1}{x^2}(x^{\frac{1}{2}} + x)$

(d) $\frac{2}{x^{-3}}(x^2 + \frac{1}{x})$ (e) $\frac{1}{\sqrt{x}}(x^2 - \sqrt{x})$ (f) $\left(x^2 + \frac{1}{x}\right)^2$

(g) $\frac{1}{x}(\sqrt{x} + x)$ (h) $\left(x + \frac{1}{\sqrt{x}}\right)^2$ (i) $x^{-2}\left(\frac{1}{x} - \sqrt[3]{x}\right)$

(j) $\frac{x^2 + 3}{x}$ (k) $\frac{\sqrt{x} - x}{x^2}$ (l) $\frac{(2x+1)^2}{x^{\frac{3}{2}}}$

INDICES

EXAM QUESTIONS

1. (a) Simplify $\frac{7a^3b^2}{a\sqrt{b}}$

(b) If $a = -1$ and $b = 4$, find the value of the expression in part (a).

2. Given that $y = 2x^{-\frac{2}{3}}$, find y when $x = 8$.

3. Simplify $x^{\frac{2}{3}}(x^{\frac{2}{3}} + x^{-\frac{2}{3}})$

4. (a) Simplify $\frac{m^5}{m^{-3}}$ (b) Evaluate $125^{-\frac{2}{3}}$

5. Express $\frac{p^5 \times 8p}{2p^{-3}}$ in its simplest form.

6. Simplify, writing your answer with a positive index: $3a^4 \times a^{-6}$

7. Simplify the fraction, giving your answer in positive index form: $\frac{x^3 \times x^4}{x^9}$

8. Simplify $\frac{a^2 \times a^5}{a^{-3}}$.

9. (a) Remove the brackets and simplify: $p^{\frac{1}{2}}(p^{\frac{5}{2}} - 2)$.

(b) Hence, or otherwise, find the value of $p^{\frac{1}{2}}(p^{\frac{5}{2}} - 2)$ when $p = 4$.

EF 1.2 CALCULATIONS USING SCIENTIFIC NOTATION

1. Rewrite these sentences with the numbers written out in full

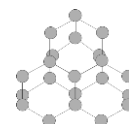
- (a) The speed of light is 3×10^8 metres per second.
- (b) The diameter of the earth is 1.268×10^4 kilometres.
- (c) A Building Society has $\pounds 2.15 \times 10^9$ in its funds.
- (d) The radius of the orbit of an electron is 5×10^{-8} mm.
- (e) A space probe reached a speed of 1.49×10^5 m.p.h.
- (f) The earth weighs 6.6×10^{21} tonnes.
- (g) A film of oil is 8×10^{-7} mm thick.

2. Use your calculator to answer the following, giving your answers in Standard Form.

- (a) $(2.2 \times 10^5) \times (4 \times 10^6)$
- (b) $(3.15 \times 10^7) \times (2.2 \times 10^8)$
- (c) $(1.8 \times 10^3) \times (2.3 \times 10^4)$
- (d) $(9.1 \times 10^6) \times (1.5 \times 10^{12})$
- (e) $(1.4 \times 10^{13}) \times (4.9 \times 10^{11})$
- (f) $(2.3 \times 10^5) \times (2.4 \times 10^7)$
- (g) $(4.25 \times 10^4) \times (2.8 \times 10^2)$
- (h) $(1.95 \times 10^{-8}) \times (3.2 \times 10^9)$
- (i) $(8.7 \times 10^5) \times (7.3 \times 10^{-10})$
- (j) $(5.05 \times 10^{-21}) \times (1.8 \times 10^{-17})$
- (k) $(2.2 \times 10^{15}) \div (4 \times 10^8)$
- (l) $(3.15 \times 10^4) \div (5 \times 10^{13})$
- (m) $(1.8 \times 10^{23}) \div (2.4 \times 10^7)$
- (n) $(1.302 \times 10^{14}) \div (1.4 \times 10^8)$
- (o) $(1.131 \times 10^{18}) \div (8.7 \times 10^{10})$
- (p) $(8.25 \times 10^5) \div (3.3 \times 10^{-7})$
- (q) $(4.25 \times 10^{-14}) \div (2.5 \times 10^{-5})$
- (r) $(8.82 \times 10^{-22}) \div (6.3 \times 10^{11})$
- (s) $(9.167 \times 10^4) \div (1.03 \times 10^{-4})$
- (t) $(6.846 \times 10^{34}) \div (6.52 \times 10^{15})$
- (u) $\frac{1.28 \times 10^6}{0.4 \times 10^2}$
- (v) $\frac{4.17 \times 10^2}{3 \times 10^{-3}}$
- (w) $\frac{18 \times 10^{-2}}{0.2 \times 10^5}$

3. Answer each of the following questions leaving your answers in standard form.

- (a) Light travels at 1.85×10^5 miles per second. How far will it travel in an hour?
- (b) The radius of the earth is 6.45×10^6 metres. What is its circumference (in km)?
- (c) If a heart beats 70 times a minute, how many times will it beat in a lifetime of 80 years?[Take all years to have 365 days]
- (d) 100 grams of water contains 2000 drops. How many drops would there be in a tank containing 1 tonne of water?
- (e) In 1 gram of carbon there are 6×10^{26} atoms. How many carbon atoms are there in 5kg of pure carbon?



4. Answer each of the following questions leaving your answers in standard form

- (a) The weight of a droplet of water is 8.7×10^{-5} grams. Calculate the weight of 10 000 droplets.
- (b) A space probe can travel at a speed of 3.6×10^6 miles per day. What distance will it travel in a week?
- (c) A biscuit factory produces 6.7×10^6 teacakes every day. How many teacakes were produced in the month of **February 2008**?
- (d) The speed of light is approximately 299 million metres per second. How far can light travel in a minute?
- (e) Last year 1.68×10^6 copies of a DVD were sold on its first day of release. If the cost of one DVD was £12, how much money was collected on that first day?
- (f) In a reality TV show there were 7.9×10^6 calls made to vote for the contestants. If each call cost 24p calculate how much the calls cost in total. Give your answer in pounds.
- (g) There are 8.64×10^4 seconds in one day. How many seconds are there in the month of April?
- (h) Organisers of the London Marathon provide enough water to give each runner 7 litres during the race. If 747 000 runners take part, how many litres of water are provided?
- (i) The exchange rate in Turkey is £1 = 2 670 000 Turkish Lira. Stephen is going on an Adriatic cruise and changes £700 into Turkish Lira. How much will he get in Lira?

1. The distance between the earth and mars is on average approximately 1.65×10^8 miles.



A spaceship has been designed to travel between the earth and mars at an average speed of 20 000 miles per hour.



How many days will the spaceship take to reach mars?

Give your answer correct to the nearest day.

2. Uranium is a radioactive isotope which has a half-life of 4.5×10^9 years. This means that only half of the original mass will be radioactive after 4.5×10^9 years.

How long will it take for the radioactivity of a piece of Uranium to reduce to **one eighth** of its original level? Give your answer in **scientific notation**.

3. The population of Scotland in June 2001 was 5.06×10^6 people.

The population of China in June 2001 was approximately 250 times larger than that of Scotland .

Calculate, correct to three significant figures, the population of China in 2001, expressing your answer in standard form.

4. The Blackbird is a two-seater high speed jet.

In December 1964 it broke a world speed record by travelling at 1.02×10^4 metres per second.



Calculate, correct to three significant figures, the distance travelled if the jet were to maintain this speed for one hour. Express your answer in scientific notation.

4.1 WORKING with TRIGONOMETRIC FUNCTIONS – BASIC GRAPHS

1. (a) With the help of a calculator, copy and complete the table below.

x°	0	30	60	90	120	150	180	210	240	270	300	330	360
$\sin x^\circ$													

- (b) Plot the points from your table.
(c) Join the points with a smooth curve.
(d) Write down the equation of the curve.

2. (a) With the help of a calculator, copy and complete the table below.

x°	0	30	60	90	120	150	180	210	240	270	300	330	360
$\cos x^\circ$													

- (b) Plot the points from your table.
(c) Join the points with a smooth curve.
(d) Write down the equation of the curve.

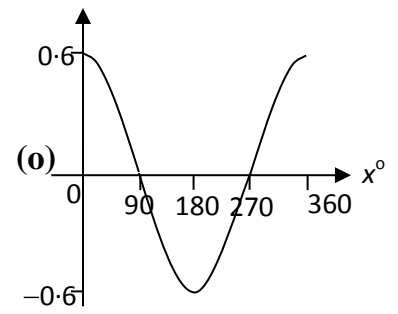
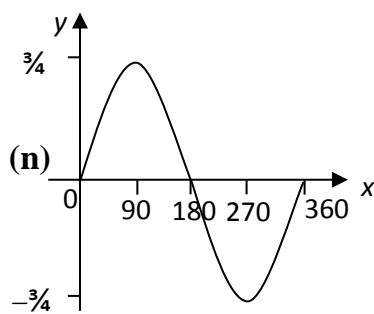
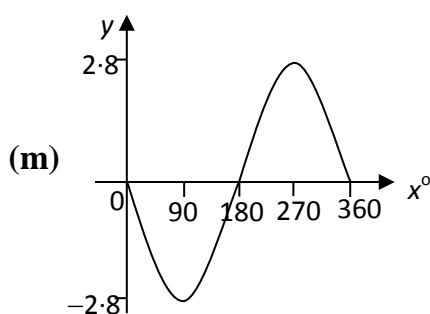
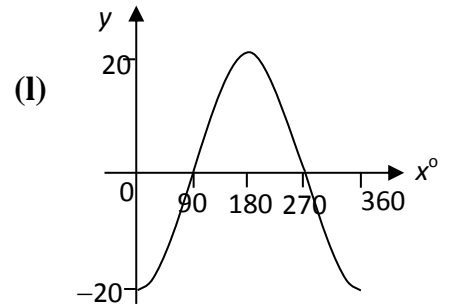
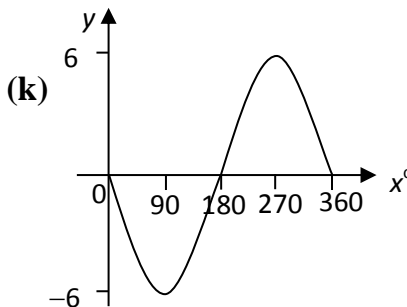
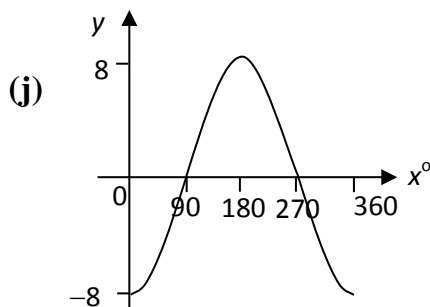
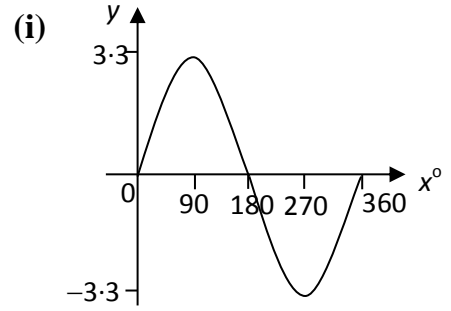
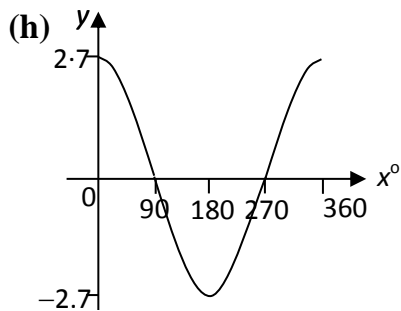
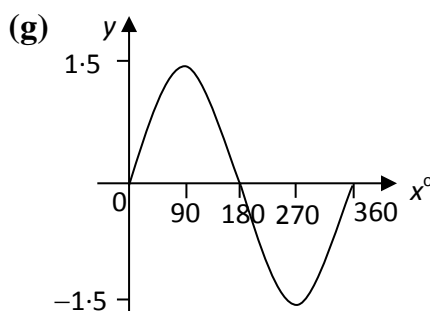
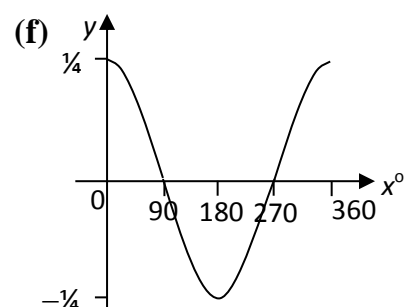
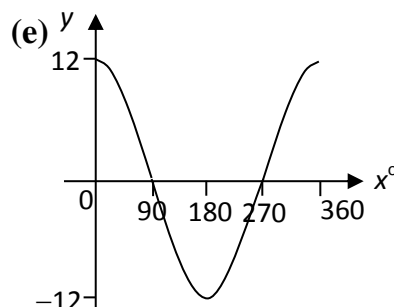
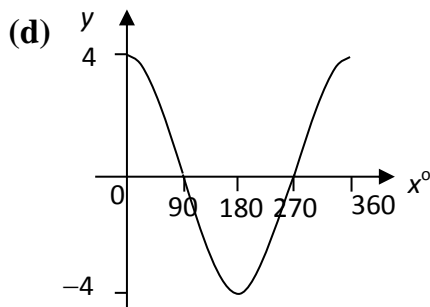
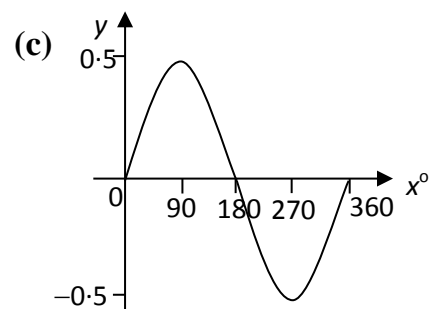
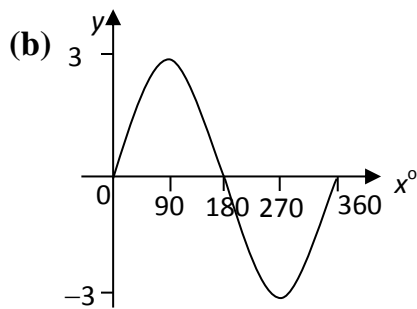
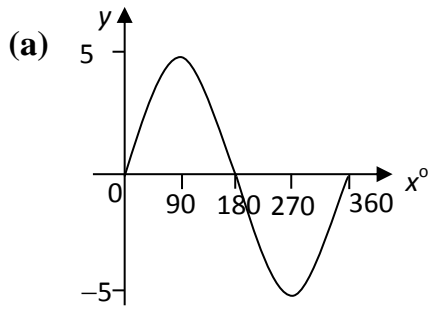
3. (a) With the help of a calculator, copy and complete the table below.

x°	0	30	60	90	120	150	180	210	240	270	300	330	360
$\tan x^\circ$													

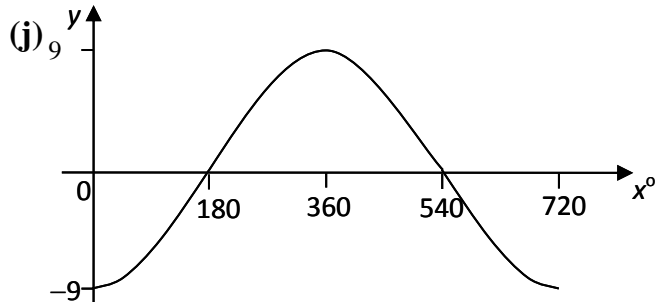
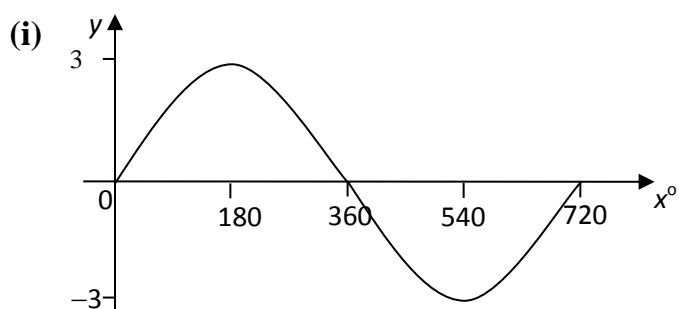
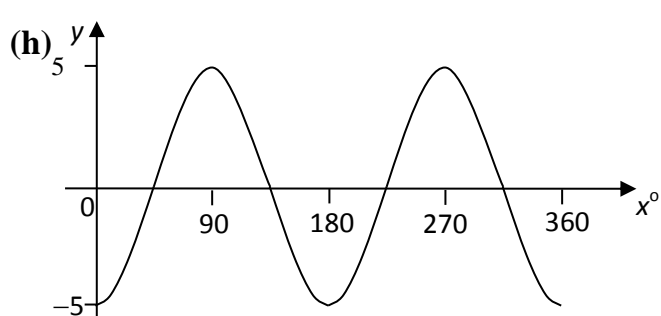
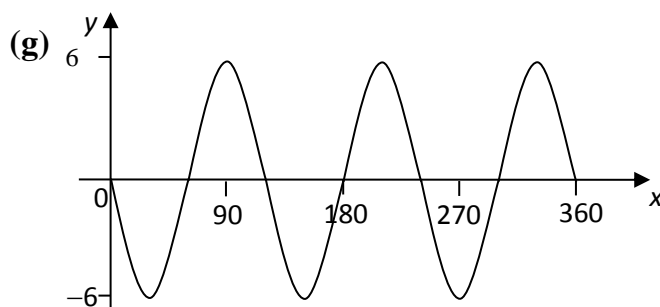
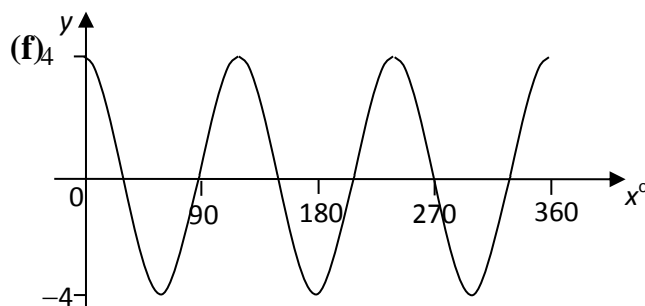
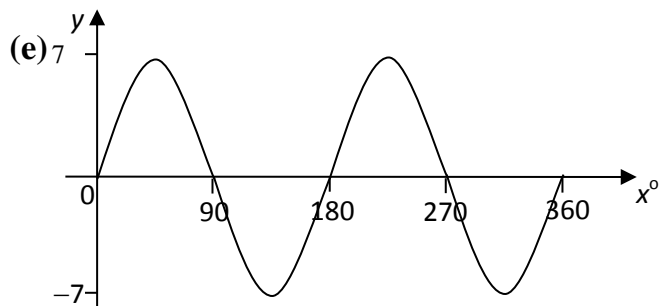
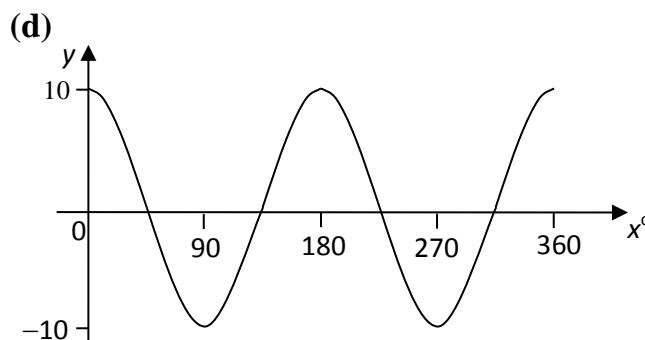
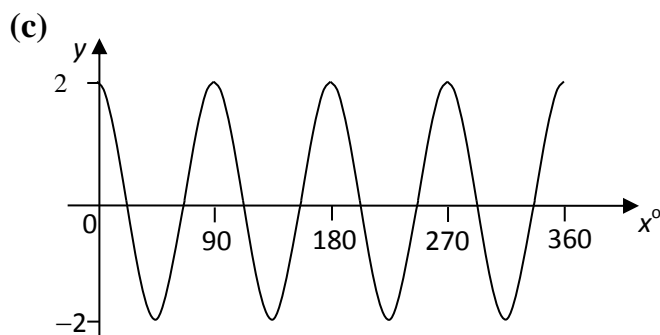
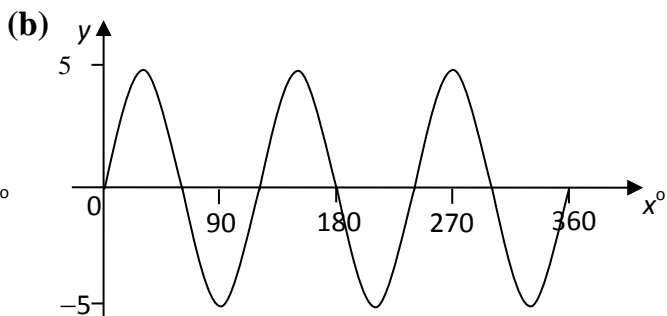
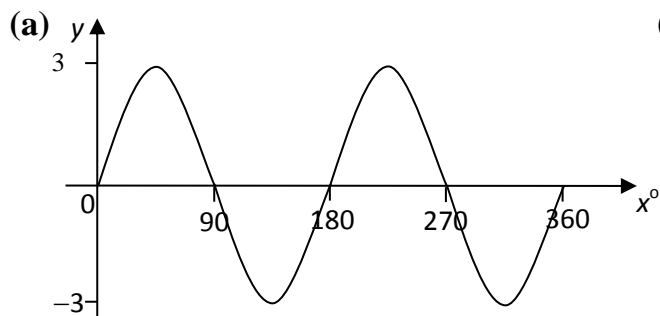
- (b) Plot the points from your table.(Be careful with the scale on the y-axis)
(c) Join the points with a smooth curve.
(d) Write down the equation of the curve.

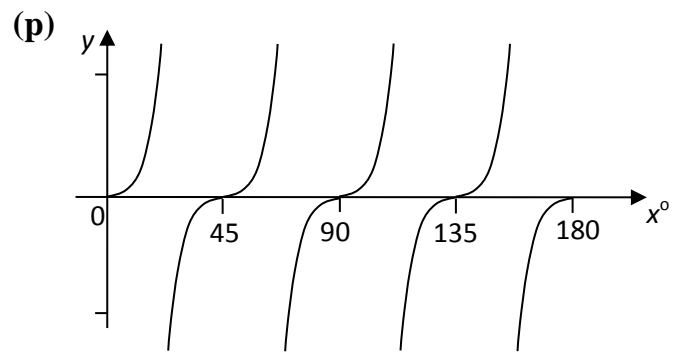
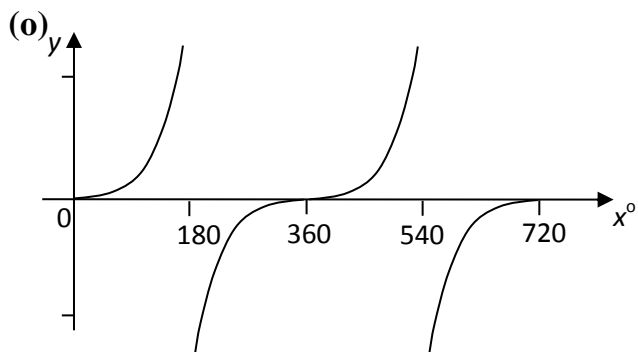
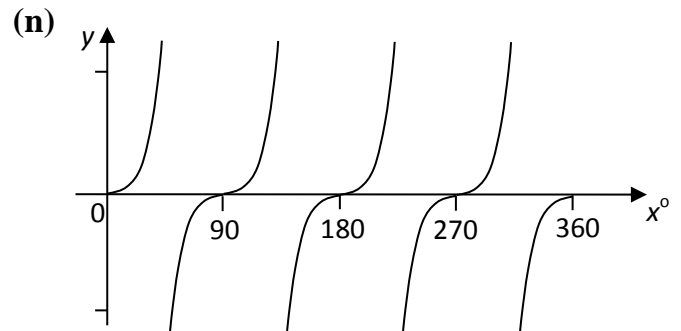
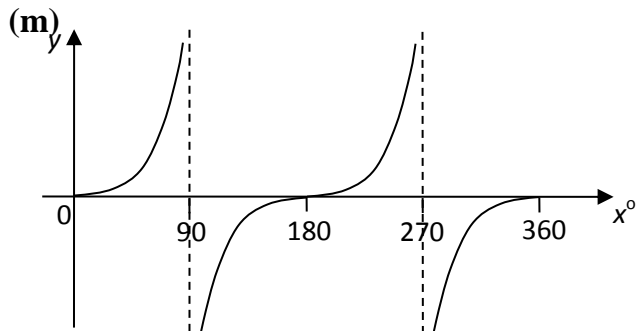
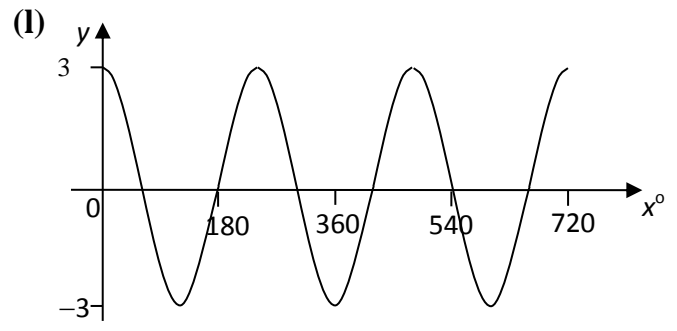
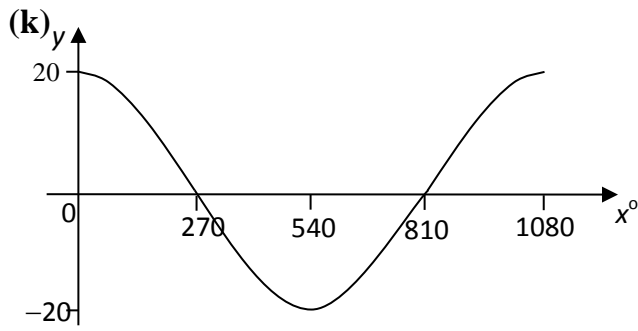
REL 4.1 WORKING with the GRAPHS of TRIGONOMETRIC FUNCTIONS (1)

1. The graphs represent the functions $a \sin x^\circ$ and $a \cos x^\circ$. Write down the equation for each.



2. The graphs represent trigonometric functions. Write down the equation for each.





3. Make sketches of the following functions, $0 \leq x < 360$, clearly marking any important points.

(a) $y = \cos x^\circ$

(b) $y = \sin x^\circ$

(c) $y = \tan x^\circ$

(d) $y = 3 \sin x^\circ$

(e) $y = 2 \cos x^\circ$

(f) $y = \sin 2x^\circ$

(g) $y = \cos 3x^\circ$

(h) $y = 2 \sin 3x^\circ$

(i) $y = 3 \cos 2x^\circ$

(j) $y = 4 \cos 3x^\circ$

(k) $y = 3 \sin \frac{1}{2}x^\circ$

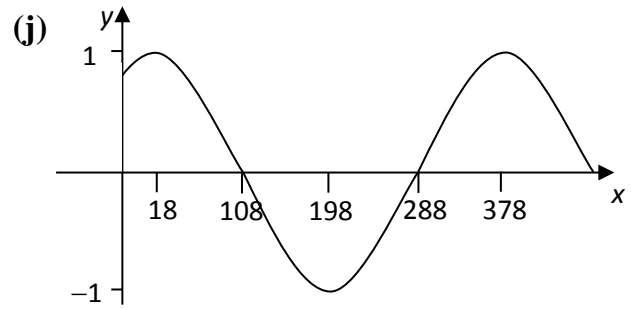
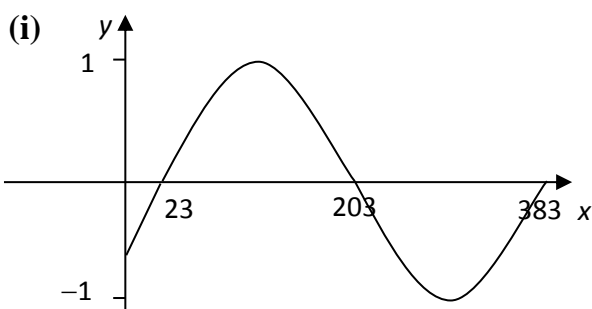
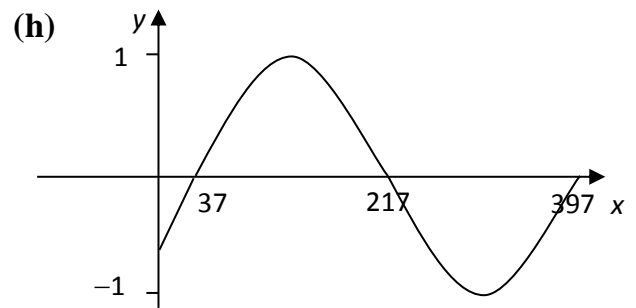
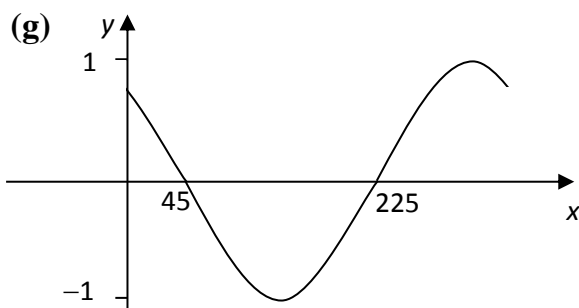
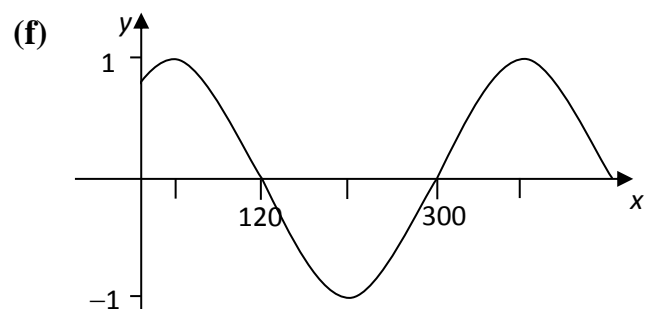
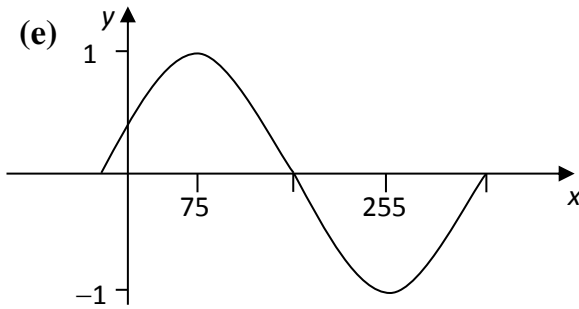
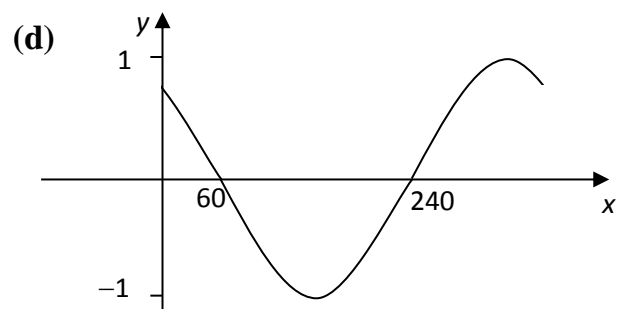
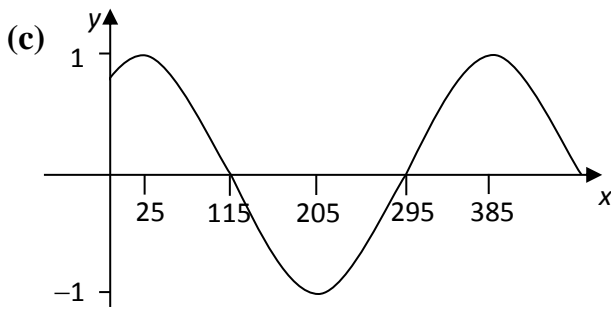
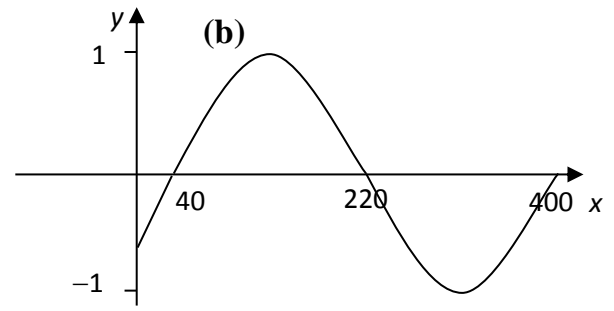
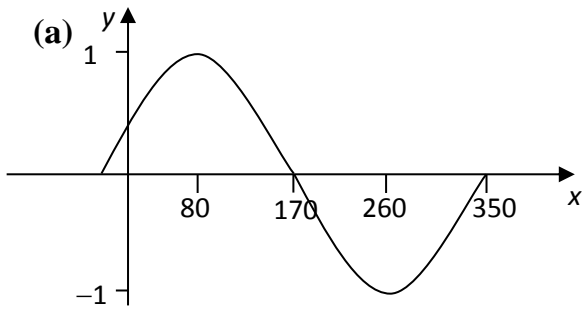
(l) $y = 5 \cos \frac{3}{2}x^\circ$

(m) $y = \tan 2x^\circ$

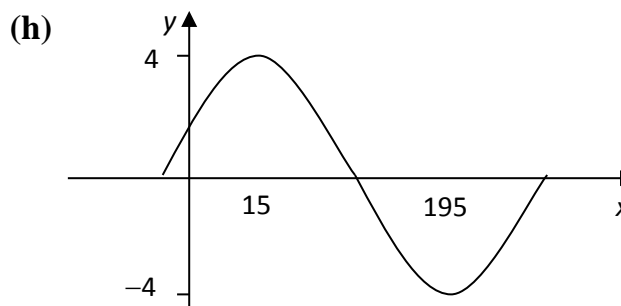
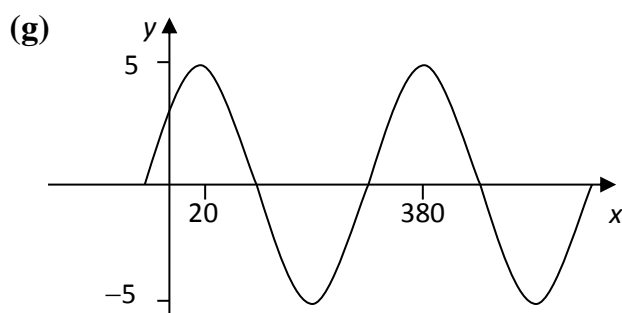
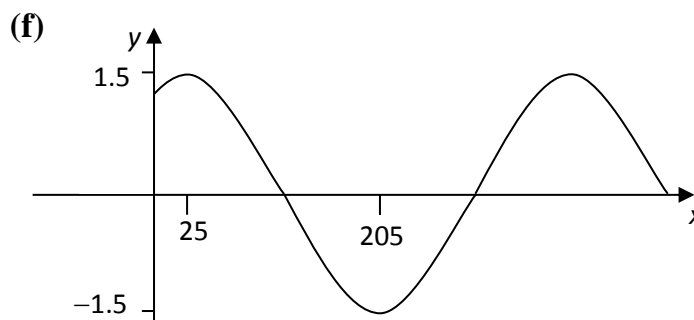
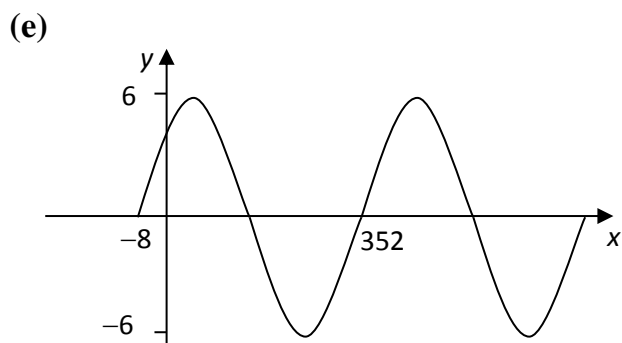
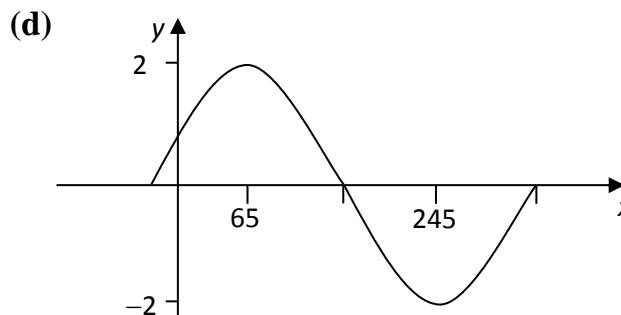
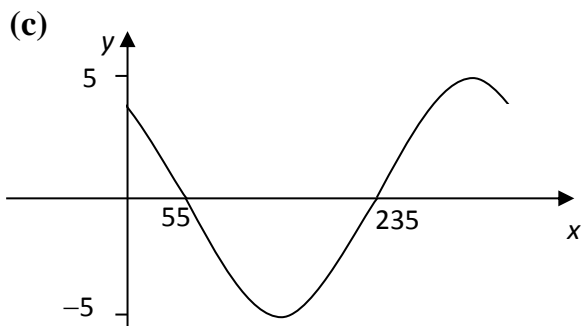
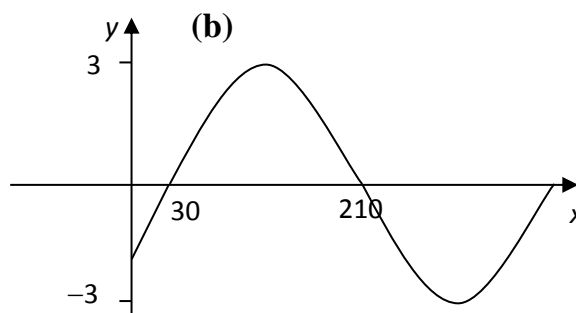
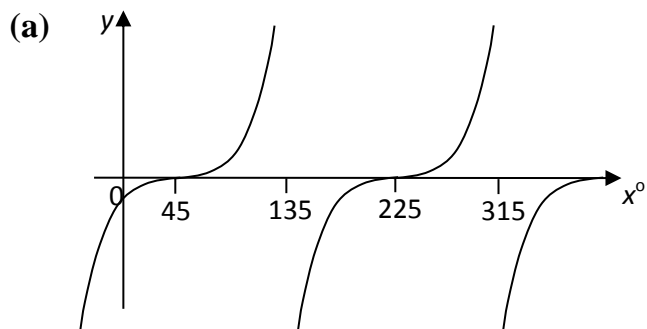
(n) $y = -2 \sin 3x^\circ$

(o) $y = -8 \cos 4x^\circ$

4. The graphs represent the functions $\sin(x \pm a)^\circ$ and $\cos(x \pm a)^\circ$. Write down the equation for each.



5. Write down the equation for each graph shown below.



6. Make a neat sketch of these trig. functions showing the important values, $0 \leq x \leq 360$.

(a) $y = \sin(x - 50)^\circ$

(b) $y = \sin(x + 30)^\circ$

(c) $y = \cos(x - 20)^\circ$

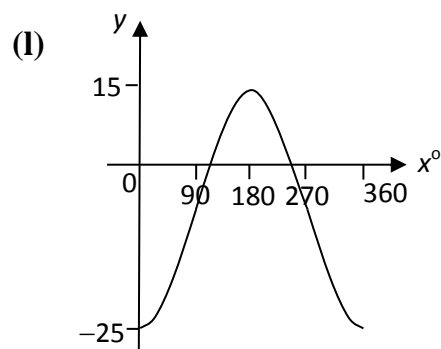
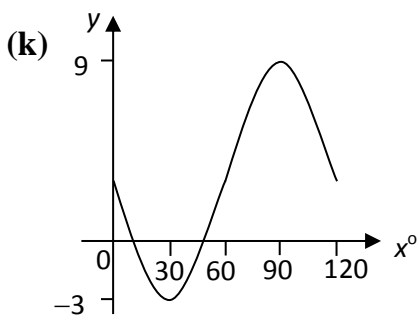
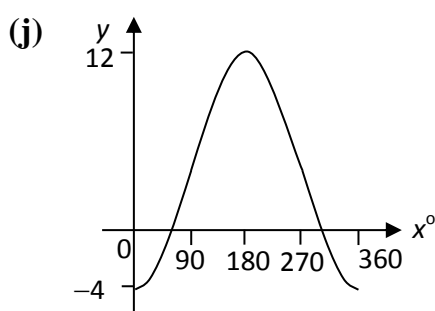
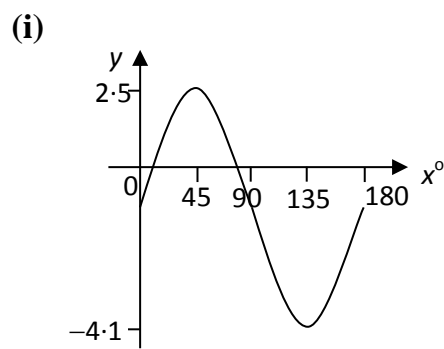
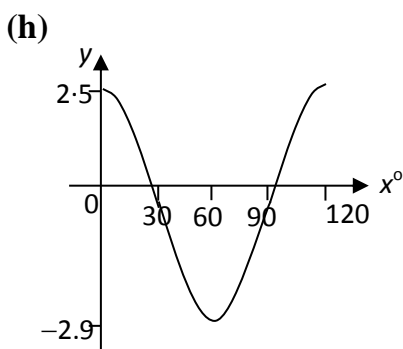
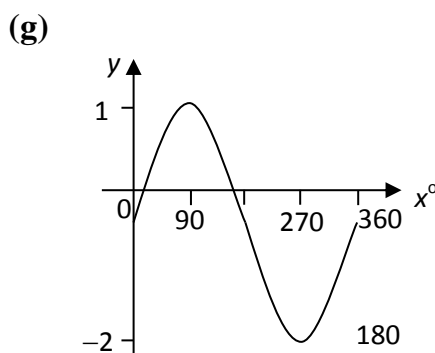
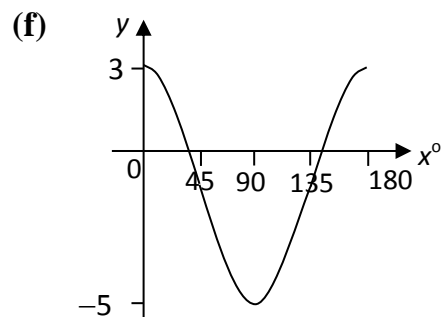
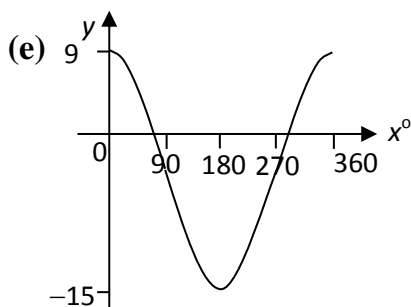
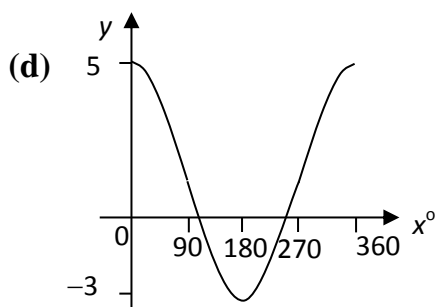
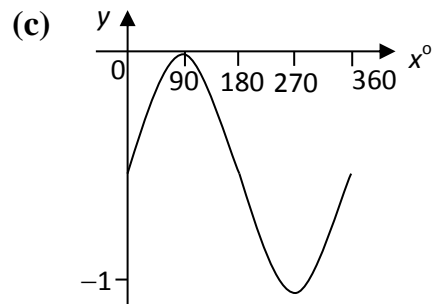
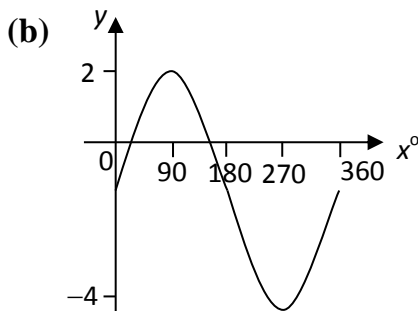
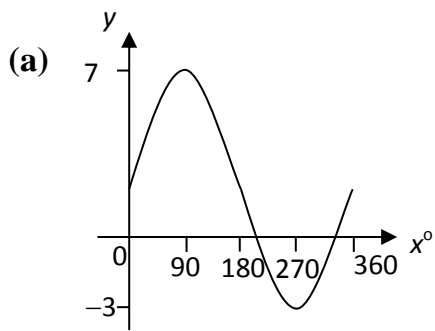
(d) $y = \cos(x + 60)^\circ$

(e) $y = \tan(x - 30)^\circ$

(f) $y = \sin(x - 45)^\circ$

REL 4.1 WORKING with the GRAPHS of TRIGONOMETRIC FUNCTIONS (2)

1. For each graph below, determine the amplitude and the equation of the graph.



2. Make sketches of the following functions, $0 \leq x < 360$, clearly marking any important points.

(a) $y = 3\cos x^\circ + 1$

(b) $y = 2\sin 3x^\circ - 2$

(c) $y = 1.5\sin 2x^\circ + 3$

(d) $y = 5\sin 2x^\circ + 1$

(e) $y = -3\cos 2x^\circ - 1$

(f) $y = -\sin x^\circ - 3$

Answers 4(a)

EF 1.1 WORKING WITH SURDS

1. (a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $5\sqrt{2}$ (d) $2\sqrt{5}$ (e) $2\sqrt{6}$ (f) $6\sqrt{3}$
(g) $2\sqrt{15}$ (h) $6\sqrt{2}$ (i) $10\sqrt{3}$ (j) $3\sqrt{3}$ (k) $4\sqrt{6}$ (l) $4\sqrt{3}$
(m) $3\sqrt{5}$ (n) $7\sqrt{2}$ (o) $3\sqrt{10}$ (p) $3\sqrt{2}$ (q) $2\sqrt{7}$ (r) $4\sqrt{5}$
(s) $4\sqrt{2}$ (t) $4\sqrt{10}$ (u) $5\sqrt{6}$ (v) $2\sqrt{11}$ (w) $3\sqrt{7}$ (x) $5\sqrt{7}$
2. (a) $10\sqrt{2}$ (b) $12\sqrt{2}$ (c) $10\sqrt{10}$ (d) $4\sqrt{3}$ (e) $12\sqrt{2}$ (f) $6\sqrt{6}$
(g) $9\sqrt{3}$ (h) $40\sqrt{3}$ (i) $12\sqrt{3}$ (j) $9\sqrt{5}$ (k) $6\sqrt{7}$ (l) $8\sqrt{5}$
3. (a) $8\sqrt{2}$ (b) $2\sqrt{7}$ (c) $3\sqrt{3}$ (d) $4\sqrt{6}$ (e) $9\sqrt{3}$ (f) $6\sqrt{6}$
(g) $3\sqrt{2}$ (h) $-6\sqrt{7}$ (i) 0 (j) $3\sqrt{5}$ (k) $-2\sqrt{3}$ (l) $11\sqrt{11}$
4. (a) $5\sqrt{3}$ (b) $2\sqrt{2}$ (c) $\sqrt{2}$ (d) $8\sqrt{2}$ (e) $6\sqrt{5}$ (f) $5\sqrt{6}$
(g) $3\sqrt{5}$ (h) $7\sqrt{10}$ (i) $3\sqrt{2}$ (j) $-\sqrt{3}$ (k) $10\sqrt{3}$ (l) $7\sqrt{5}$
(m) $8\sqrt{3}$ (n) $2\sqrt{2}$ (o) $\sqrt{2}$ (p) $8\sqrt{2}$ (q) $6\sqrt{5}$ (r) $5\sqrt{6}$
(s) $7\sqrt{2}$ (t) $8\sqrt{3}$ (u) $4\sqrt{2}$
5. (a) 5 (b) 2 (c) 11 (d) a (e) 6 (f) c
(g) k (h) $3\sqrt{2}$ (i) 4 (j) $2\sqrt{3}$ (k) $\sqrt{15}$ (l) \sqrt{xy}
(m) 4 (n) 6 (o) 10 (p) 8 (q) \sqrt{ab} (r) $\sqrt{10x}$
(s) \sqrt{pq} (t) $\sqrt{6k}$ (u) $2\sqrt{5}$ (v) $6\sqrt{2}$ (w) $5\sqrt{2}$ (x) $6\sqrt{2}$
(y) $2\sqrt{15}$ (z) $4\sqrt{2}$
6. (a) 6 (b) 30 (c) $6\sqrt{14}$ (d) 24 (e) $3\sqrt{10}$ (f) $18\sqrt{2}$
(g) $16\sqrt{6}$ (h) $15\sqrt{15}$
7. (a) 2 (b) $\frac{3}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) 2 (f) $\frac{1}{2}$
(g) $\frac{3}{2}$ (h) $\frac{5}{3}$ (i) $\frac{1}{2}$ (j) $\frac{1}{3}$ (k) 6 (l) $\frac{10}{3}$
(m) $2\sqrt{2}$ (n) $\frac{1}{2\sqrt{2}}$ (o) $\sqrt{14}$ (p) $\frac{1}{\sqrt{5}}$
8. (a) $\sqrt{2} - 2$ (b) $3 + \sqrt{3}$ (c) $5 - \sqrt{5}$ (d) $5\sqrt{2} + 2$
(e) $3\sqrt{2} + 2\sqrt{3}$ (f) $4\sqrt{6} + 2\sqrt{3}$ (g) $3\sqrt{2} - 4\sqrt{6}$ (h) $5 + 2\sqrt{5}$
(i) $48 - 16\sqrt{3}$ (j) $4 + 8\sqrt{2}$ (k) $12 + 12\sqrt{2}$ (l) $15\sqrt{10}$

- (m)** $\sqrt{6} + \sqrt{3}$ **(n)** 6 **(o)** $\sqrt{6} + 3\sqrt{2}$ **(p)** $3\sqrt{5} - 5$
- 9. (a)** $2\sqrt{2} - 1$ **(b)** $6 - 2\sqrt{5}$ **(c)** $16 + 11\sqrt{2}$ **(d)** 2
(e) -1 **(f)** 1 **(g)** $10 - 13\sqrt{2}$ **(h)** $10 + 3\sqrt{8}$
(i) $12 + 7\sqrt{6}$ **(j)** $11 + 6\sqrt{2}$ **(k)** $5 + 2\sqrt{6}$ **(l)** $13 - 4\sqrt{3}$
(m) $30 - 4\sqrt{14}$ **(n)** $37 - 20\sqrt{3}$ **(o)** -2 **(p)** $8 + 2\sqrt{7}$
(q) $8 + 4\sqrt{3}$ **(r)** -1
- 10. (a)** $\frac{\sqrt{2}}{2}$ **(b)** $\frac{\sqrt{3}}{3}$ **(c)** $\frac{\sqrt{5}}{5}$ **(d)** $2\sqrt{3}$ **(e)** $2\sqrt{5}$ **(f)** $\frac{2\sqrt{3}}{3}$
(g) $\frac{3\sqrt{5}}{5}$ **(h)** $10\sqrt{2}$ **(i)** $\sqrt{2}$ **(j)** $4\sqrt{3}$ **(k)** $\frac{\sqrt{6}}{2}$ **(l)** $\frac{4\sqrt{5}}{5}$
(m) $5\sqrt{2}$ **(n)** $5\sqrt{7}$
- 11. (a)** $\frac{\sqrt{5}}{10}$ **(b)** $\frac{2\sqrt{2}}{5}$ **(c)** $\frac{\sqrt{2}}{2}$ **(d)** $\frac{2\sqrt{6}}{5}$ **(e)** $\frac{4\sqrt{2}}{3}$ **(f)** $\frac{4\sqrt{5}}{7}$
(g) $\frac{5\sqrt{10}}{3}$ **(h)** $\frac{5\sqrt{2}}{3}$
- 12. (a)** $\frac{\sqrt{6}}{2}$ **(b)** $\frac{\sqrt{10}}{5}$ **(c)** 2 **(d)** $\sqrt{6}$
(e) $\frac{1}{2}$ **(f)** $\frac{\sqrt{6}}{6}$ **(g)** $\sqrt{3}$ **(h)** $\frac{2\sqrt{3}}{3}$
(i) $\frac{\sqrt{10}}{2}$ **(j)** $\frac{\sqrt{22}}{2}$ **(k)** $\frac{\sqrt{21}}{3}$ **(l)** $\frac{\sqrt{65}}{5}$
(m) $\frac{2}{3}$ **(n)** $\frac{\sqrt{6}}{3}$ **(o)** $\frac{\sqrt{15}}{3}$ **(p)** $\frac{4\sqrt{15}}{15}$
(q) $\frac{\sqrt{3}}{3}$ **(r)** $\sqrt{5}$ **(s)** $\frac{1}{2}$ **(l)** $\frac{\sqrt{10}}{2}$
- 13. (a)** $\frac{\sqrt{2}}{10}$ **(b)** $2\sqrt{3}$ **(c)** $\frac{\sqrt{2}}{2}$ **(d)** $\frac{3\sqrt{5}}{10}$
(e) $\sqrt{2}$ **(f)** $\frac{\sqrt{2}}{2}$ **(g)** $\frac{5\sqrt{3}}{3}$ **(h)** $\frac{3\sqrt{2}}{10}$
(i) $\frac{\sqrt{2}}{2}$ **(j)** $\frac{\sqrt{2}}{3}$ **(k)** $\frac{\sqrt{3}}{2}$ **(l)** $\frac{2}{3}$
- 14. (a)** $\sqrt{2+1}$ **(b)** $\frac{\sqrt{5}-1}{4}$ **(c)** $-12(2 + \sqrt{3})$ **(d)** $-(1 + \sqrt{2})$

(e)	$-\frac{1}{2}(1-\sqrt{3})$	(f)	$\frac{3(\sqrt{5}+1)}{4}$	(g)	$-(\sqrt{2}-2)$	(h)	$-\frac{3}{2}(2+\sqrt{6})$
(i)	$\frac{5(3-\sqrt{2})}{7}$	(j)	$-2(1+\sqrt{3})$	(k)	$\frac{\sqrt{7}+2}{3}$	(l)	$\sqrt{3}+\sqrt{2}$
(m)	$6(\sqrt{3}-\sqrt{2})$	(n)	$\frac{3}{2}(\sqrt{10}+\sqrt{2})$	(o)	$-3(\sqrt{5}-\sqrt{6})$	(p)	$\frac{14(9+\sqrt{2})}{79}$

SURDS

- (a) $3\sqrt{5}$ (b) $\sqrt{3}$ (c) $6\sqrt{5}$ (d) $2\sqrt{2}$
- (a) 10 (b) -2 (c) 6 (d) $4\sqrt{2}$
- (a) $4\sqrt{3}$ (b) 8 (c) $4\sqrt{15}$
- (a) 2 cm^2 (b) $2\sqrt{3}\text{ cm}$
- (a) 3 (b) 11
- $8\sqrt{2}$
- (a) $\sqrt{3}a$ (b) $\sqrt{3}a^2$
- Proof
- Proof

PROBLEMS

EF 1.2 INDICES

- (a) 3^6 (b) 2^4 (c) 10^7 (d) 8^8 (e) 7^7 (f) 5^8
 (g) 9^8 (h) 6^{13} (i) x^8 (j) c^{11} (k) a^{14} (l) y^{10}
 (m) b^{40} (n) p^{10} (o) d^6 (p) q^{20} (q) t^{10} (r) f^7
 (s) k^{13} (t) z^{100} (u) x^{80} (v) y^{20} (w) a^{90} (x) b^1
- (a) 2^5 (b) 5^2 (c) 12^3 (d) 7^7 (e) 20^4 (f) 8^4
 (g) 3^{15} (h) 4^2 (i) x^5 (j) a^4 (k) y^{10} (l) b^3
 (m) p (n) 1 (o) q^6 (p) d^3 (q) x^6 (r) a^6
 (s) m^{13} (t) 1 (u) d^8 (v) y^{90} (w) t^{99} (x) w^{10}
- (a) 3^8 (b) 8^4 (c) 10^6 (d) 2^{10} (e) 4^{15} (f) 1
 (g) 12^9 (h) 5^{25} (i) x^8 (j) y^{40} (k) a^{21} (l) m^{16}
 (m) b^{18} (n) p^{15} (o) k^{100} (p) 1
- (a) $4b^2$ (b) $343a^3$ (c) $81x^4$ (d) $32y^5$ (e) a^4b^4 (f) x^7y^7

- (g)** w^5z^5 **(h)** s^3t^3 **(i)** p^3q^6 **(j)** x^8y^2 **(k)** $a^{10}b^{15}$ **(l)** $36a^{10}$
(m) $1000x^6$ **(n)** $32c^{20}$ **(o)** $27a^3b^6$ **(p)** $16m^4k^2$
5. **(a)** $10a^8$ **(b)** $63x^9$ **(c)** $3p^3$ **(d)** $5b^6$
(e) $24y^7$ **(f)** $80q^{10}$ **(g)** $8c^7$ **(h)** $8z^4$
(i) $k^5 + k^7$ **(j)** $m^7 - m^8$ **(k)** $2x^7 + 6x^6$ **(l)** $10a^7 - 15a^8$
(m) x^3 **(n)** m^{14} **(o)** $10c^4$ **(p)** $6q^3$
(q) $3xy^{14}$ **(r)** $4a^{10}b^{28}$ **(s)** $4p^3$ **(t)** $\frac{8}{3}a^2b^{12}$
6. **(a)** 1 **(b)** 1 **(c)** 1 **(d)** 1 **(e)** 1 **(f)** 1
(g) 1 **(h)** 1 **(i)** 1 **(j)** 1 **(k)** 1 **(l)** 1
7. **(a)** $\frac{1}{3^2}$ **(b)** $\frac{1}{5^4}$ **(c)** $\frac{1}{2^6}$ **(d)** $\frac{1}{10^3}$ **(e)** $\frac{1}{4^5}$ **(f)** $\frac{1}{200^7}$
(g) $\frac{1}{a^5}$ **(h)** $\frac{1}{x^2}$ **(i)** $\frac{1}{p^7}$ **(j)** $\frac{1}{y^{10}}$ **(k)** $\frac{2}{b^3}$ **(l)** $\frac{10}{q^x}$
(m) x^3 **(n)** w^5 **(o)** $3a^2$ **(p)** $10c^8$ **(q)** $\frac{2}{3}t$ **(r)** $\frac{5}{4}y^3$
8. **(a)** 3^{-2} **(b)** 6^{-9} **(c)** 5^{-4} **(d)** 2^{-7} **(e)** 10^{-3} **(f)** 4^{-4}
(g) x^{-3} **(h)** a^{-5} **(i)** p^{-4} **(j)** y^{-10} **(k)** q^{-6} **(l)** c^{-8}
9. **(a)** m^{-2} **(b)** x^5 **(c)** p^{-3} **(d)** a^{-8} **(e)** y^{-12} **(f)** c^{-15}
(g) q^{-15} **(h)** w^8 **(i)** $20b$ **(j)** 27 **(k)** $2k^5$ **(l)** $1.5d^{-3}$
(m) $x^5 + x$ **(n)** $p - p^{-11}$ **(o)** $6a^6 + 9a^3$ **(p)** $2m^{-5} - 5m^4$
(q) v^{10} **(r)** $6h^{-1}$ **(s)** $6c^5$ **(t)** 10
10. **(a)** 2 **(b)** 2 **(c)** 6 **(d)** 9 **(e)** 4 **(f)** 10
(g) 5 **(h)** 27 **(i)** 25 **(j)** 8 **(k)** 6 **(l)** $\frac{1}{2}$
(m) $\frac{1}{2}$ **(n)** $\frac{1}{4}$ **(o)** $\frac{1}{3}$ **(p)** $\frac{1}{9}$ **(q)** $\frac{1}{64}$ **(r)** $\frac{1}{100}$
(s) $\frac{1}{64}$ **(t)** $\frac{1}{16}$ **(u)** 16 **(v)** -2 **(w)** 16 **(x)** $\frac{1}{1000}$
(y) 2 **(z)** $\frac{1}{16}$

11. (a) x^3 (b) p^2 (c) a^6 (d) $\frac{1}{y^6}$ (e) $\frac{1}{q^2}$ (f) $\frac{1}{k^{\frac{2}{5}}}$
 (g) g^2 (h) $\frac{1}{m^8}$ (i) c^6 (j) $\frac{1}{h^{\frac{5}{2}}}$ (k) $\frac{1}{z^3}$ (l) $\frac{1}{b^{12}}$
 (m) 1 (n) y (o) d^2 (p) s^3 (q) $12x$ (r) 12
 (s) $10x$ (t) $6x^{\frac{1}{3}}$ (u) 1 (v) $2x$ (w) $4x^{\frac{1}{3}}$ (x) $\frac{3}{2x^{\frac{1}{3}}}$

12. (a) \sqrt{x} (b) $\sqrt[3]{x}$ (c) $\sqrt[4]{a}$ (d) $\sqrt[3]{y^2}$
 (e) $\sqrt[4]{b^3}$ (f) $\sqrt[3]{x^5}$ (g) $\sqrt[5]{c^3}$ (h) $\sqrt[5]{a^4}$
 (i) $\frac{1}{\sqrt[3]{c}}$ (j) $\frac{1}{\sqrt{z}}$ (k) $\frac{1}{\sqrt[3]{m^2}}$ (l) $\frac{1}{\sqrt[5]{k^3}}$
 (m) $\frac{1}{\sqrt[3]{p^4}}$ (n) $\frac{1}{\sqrt[3]{x^5}}$ (o) $\frac{1}{\sqrt[5]{w^4}}$ (p) $\frac{1}{\sqrt[7]{d^2}}$

13. (a) $x^{\frac{1}{2}}$ (b) $a^{\frac{1}{3}}$ (c) $y^{\frac{3}{2}}$ (d) $z^{\frac{2}{3}}$ (e) $c^{\frac{2}{3}}$ (f) $x^{\frac{3}{4}}$
 (g) $p^{\frac{5}{3}}$ (h) $m^{\frac{2}{5}}$ (i) $a^{-\frac{1}{2}}$ (j) $z^{-\frac{1}{3}}$ (k) $x^{-\frac{4}{3}}$ (l) $a^{-\frac{1}{5}}$
 (m) $b^{-\frac{2}{3}}$ (n) $m^{-\frac{3}{5}}$ (o) $y^{-\frac{1}{4}}$ (p) $c^{-\frac{5}{3}}$

14. (a) $x^{\frac{9}{2}} + x^{\frac{1}{2}}$ (b) $x - x^{\frac{3}{2}}$ (c) $x^{-\frac{3}{2}} + x^{-1}$
 (d) $2x^5 + 2x^2$ (e) $x^{\frac{3}{2}} - 1$ (f) $x^4 + 2x + \frac{1}{x^2}$
 (g) $x^{-\frac{1}{2}} + 1$ (h) $x^2 + 2x^{\frac{1}{2}} + x^{-1}$ (i) $x^{-3} - x^{-\frac{5}{3}}$
 (j) $x + 3x^{-1}$ (k) $x^{-\frac{3}{2}} - x^{-1}$ (l) $4x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$

INDICES EXAM QUESTIONS

1. (a) $7a^2b^{\frac{3}{2}}$ (b) 56 2. $\frac{1}{2}$ 3. $x^{\frac{4}{3}} + 1$
 4. (a) m^8 (b) $\frac{1}{25}$ 5. $4p^9$ 6. $\frac{3}{a^2}$ 7. $\frac{1}{x^2}$ 8. a^{10}
 9. (a) $p^3 - 2p^{\frac{1}{2}}$ (b) 60

EF 1.2 CALCULATIONS USING SCIENTIFIC NOTATION

1. (a) The speed of light is 300 000 000 metres per second.
(b) The diameter of the earth is 12 680 kilometres.
(c) A Building Society has £2 150 000 000 in its funds.
(d) The radius of the orbit of an electron is 0.000 000 05 mm.
(e) A space probe reached a speed of 149 000 m.p.h.
(f) The earth weighs 6 600 000 000 000 000 000 000 tonnes.
(g) A film of oil is 0.000 000 08 mm thick.
2. (a) 8.8×10^{11} (b) 6.93×10^{15} (c) 4.14×10^7 (d) 1.365×10^{19}
(e) 6.86×10^{24} (f) 5.52×10^{12} (g) 1.19×10^7 (h) 6.24×10
(i) 6.351×10^{-4} (j) 9.09×10^{-38} (k) 5.5×10^6 (l) 6.3×10^{-10}
(m) 7.5×10^{15} (n) 9.3×10^5 (o) 1.3×10^7 (p) 2.5×10^{12}
(q) 1.7×10^{-9} (r) 1.4×10^{-33} (s) 8.9×10^8 (t) 1.05×10^{19}
(u) 3.2×10^4 (v) 1.39×10^5 (w) 9×10^{-6}
3. (a) 6.66×10^8 (b) 4.0506×10^4 (c) 2.94336×10^9
(d) 2×10^7 (e) 3×10^{30}
4. (a) 8.7×10^{-1} grams. (b) 2.52×10^7 (c) 1.943×10^8
(d) 1.794×10^{10} (e) $£2.016 \times 10^7$ (f) $£1.896 \times 10^6$
(g) 2.592×10^6 (h) 5.229×10^6 (i) 1.869×10^9

SCIENTIFIC NOTATION

EXAM QUESTIONS

1. 344 days 2. 1.35×10^{10} years 3. 1.27×10^9 4. 3.672×10^7